

# First-order Monadic Equivalences

## Notation:

“ $A(x)$ ”, “ $B(x)$ ”, “ $C(x)$ ”, ... stand for (open) schemata with the free variable  $x$ .

“ $P$ ”, “ $Q$ ”, “ $R$ ”, ... stand for (closed) schemata with *no* free variables.

Unless otherwise specified, quantifiers only apply to their *nearest* schema. Thus, “ $\forall x F(x) . G(x)$ ” is short for “ $\forall x (F(x)) . G(x)$ ”.

## **Negation Rules:**

$$\forall x A(x) \Leftrightarrow \neg \exists x \neg A(x)$$

$$\exists x A(x) \Leftrightarrow \neg \forall x \neg A(x)$$

$$\neg \forall x A(x) \Leftrightarrow \exists x \neg A(x)$$

$$\neg \exists x A(x) \Leftrightarrow \forall x \neg A(x)$$

## **Conjunction & Disjunction:**

$$\forall x (A(x) . B(x)) \Leftrightarrow \forall x A(x) . \forall x B(x)$$

$$\exists x (A(x) \vee B(x)) \Leftrightarrow \exists x A(x) \vee \exists x B(x)$$

$$\forall x (A(x) \vee B(x)) \Leftrightarrow \forall x A(x) \vee \forall x B(x)$$

$$\exists x (A(x) . B(x)) \Rightarrow \exists x A(x) . \exists x B(x)$$

## **Quantifying on Closed Schemata:**

$$\forall x P \Leftrightarrow P$$

$$\exists x P \Leftrightarrow P$$

$$\forall x (A(x) \vee P) \Leftrightarrow \forall x A(x) \vee P$$

$$\exists x (A(x) . P) \Leftrightarrow \exists x A(x) . P$$

## **Quantifiers and Conditionals:**

$$P \supset \forall x A(x) \Leftrightarrow \forall x (P \supset A(x))$$

$$P \supset \exists x A(x) \Leftrightarrow \exists x (P \supset A(x))$$

$$\forall x A(x) \supset P \Leftrightarrow \exists x (A(x) \supset P)$$

$$\exists x A(x) \supset P \Leftrightarrow \forall x (A(x) \supset P)$$