First-order Monadic Equivalences

Notation:
"$A(x)$", "$B(x)$", "$C(x)$", . . . stand for (open) schemata with the free variable x.
"P", "Q", "R", . . . stand for (closed) schemata with *no* free variables.
Unless otherwise specified, quantifiers only apply to their *nearest* schema. Thus, "$\forall x\, F(x) \,.\, G(x)$" is short for "$\forall x\ \ (F(x))\ \ .\, G(x)$".

Negation Rules:
$\forall x\ \ A(x) \Leftrightarrow -\exists x\ \ -A(x)$
$\exists x\ \ A(x)\ \ \Leftrightarrow -\forall x\ \ -A(x)$

$-\forall x\ \ A(x) \Leftrightarrow \exists x\ \ -A(x)$
$-\exists x\ \ A(x) \Leftrightarrow \forall x\ \ -A(x)$

Conjunction & Disjunction:
$\forall x\ \ (A(x) \,.\, B(x)) \Leftrightarrow \forall x\ \ A(x) \,.\, \forall x\ \ B(x)$
$\exists x\ \ (A(x) \vee B(x)) \Leftrightarrow \exists x\ \ A(x) \vee \exists x\ \ B(x)$

$\forall x\ \ (A(x) \vee B(x)) \Leftarrow \forall x\ \ A(x) \vee \forall x\ \ B(x)$
$\exists x\ \ (A(x) \,.\, B(x)) \Rightarrow \exists x\ \ A(x) \,.\, \exists x\ \ B(x)$

Quantifying on Closed Schemata:
$\forall x\ \ P \Leftrightarrow P$
$\exists x\ \ P \Leftrightarrow P$

$\forall x\ \ (A(x) \vee P) \Leftrightarrow \forall x\ \ A(x) \vee P$
$\exists x\ \ (A(x) \,.\, P) \Leftrightarrow \exists x\ \ A(x) \,.\, P$

Quantifiers and Conditionals:
$P \supset \forall x\ \ A(x) \Leftrightarrow \forall x\ \ (P \supset A(x))$
$P \supset \exists x\ \ A(x) \Leftrightarrow \exists x\ \ (P \supset A(x))$

$\forall x\ \ A(x) \supset P \Leftrightarrow \exists x\ \ (A(x) \supset P)$
$\exists x\ \ A(x) \supset P \Leftrightarrow \forall x\ \ (A(x) \supset P)$