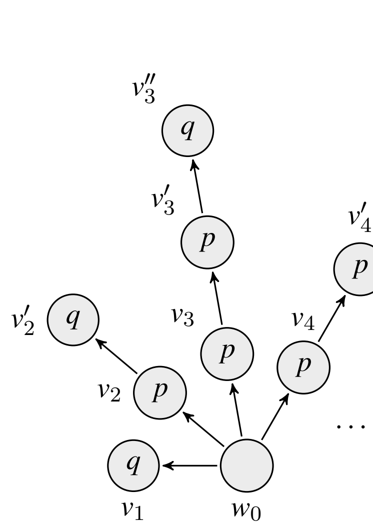
Selection Method Example

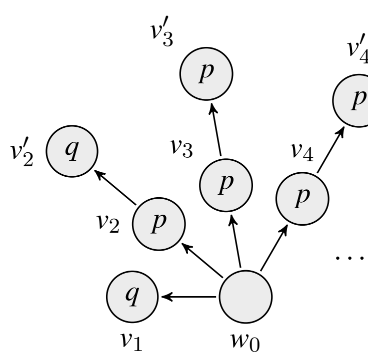
Phil 143 Handout

Consider the following model $\mathcal{M}$:



We will demonstrate the selection method using the formula $\Diamond\Diamond p \to \Diamond q$. Usually, the first step is to perform a tree unraveling on the original model. However, this model is already "unraveled" so-to-speak, so we can move on to the next step.

The next step is to cut off the tree at the modal depth of the formula in question. Since the modal depth of the formula is 2, we just need to chop off all the nodes that are more than 2 steps away from w_0. The result is the following model $\mathcal{N}$:



Finally, we need to "prune" the width of the tree by selecting the worlds we want in our final model. To do this, we first determine what the $\Diamond$-subformula of $\Diamond\Diamond p \to \Diamond q$ are. In our case, there are only three: $\Diamond\Diamond p$, $\Diamond p$, and $\Diamond q$.

Now we start selecting worlds. To select worlds, we first determine which of these $\Diamond$-subformulas is true at w_0. For each one that's true at w_0, we'll select a world from among the worlds w_0 can see that witnesses the truth of that $\Diamond$-subformula.

It's straightforward to check that:

- $\mathcal{N}, w_0 \models \Diamond\Diamond p$
- $\mathcal{N}, w_0 \models \Diamond p$
- $\mathcal{N}, w_0 \models \Diamond q$

So now we must pick a world that witnesses the truth of $\Diamond\Diamond p$ for w_0. That is, we must pick a world that w_0 can see such that $\Diamond p$ is true. We have an infinite number of choices: $v_3, v_4, v_5, \ldots$. Let's keep things simple and select v_3.

Next, we must pick a world that witnesses the truth of $\Diamond p$ for w_0. That is, we must pick a world that w_0 can see such that p is true. We again have an infinite number of choices: $v_2, v_2, v_4, v_5, \ldots$. But since we've already chosen v_3, we can make our life easier by selecting v_3 again.

Finally, we must pick a world that witnesses the truth of $\Diamond q$ for w_0. That is, we must pick a world that w_0 can see such that q is true. In this case, we have only one choice: v_1. So we select v_1.

Now we're not done. We need to continue the selection process *for all of our newly selected worlds*, namely v_1 and v_3. It's straightforward to check the following:

- $\mathcal{N}, v_1 \not\models \Diamond\Diamond p$
- $\mathcal{N}, v_1 \not\models \Diamond p$
- $\mathcal{N}, v_1 \not\models \Diamond q$
- $\mathcal{N}, v_3 \not\models \Diamond\Diamond p$
- $\mathcal{N}, v_3 \models \Diamond p$
- $\mathcal{N}, v_3 \not\models \Diamond q$

Thankfully, since the only subformula satisfied is $\Diamond p$ at v_3, we just need to pick a world that witnesses the truth of $\Diamond p$ for v_3. Since v_3 can only see one world, namely v_3', and since this world satisfies p, we only have one choice: v_3'.

Now, you we need to see whether we need to select any worlds for v_3'. Fortunately, we don't, since v_3' can't see any worlds in $\mathcal{N}$ anyway (so there's no way it can satisfy any $\Diamond$-subformula). Thus, our final selected model looks like: