Axiomatic Proofs

Phil 143 Worksheet

1. Produce a formal proof in **K** for the following formulas:

   (a) $\Box p \to \Box(q \to p)$

   (b) $(\Diamond p \to \Box q) \to (\Box p \to \Box q)$

   (c) $\Box(p \to q) \to (\Diamond p \to \Diamond q)$

   (d) $\Diamond(p \to q) \to (\Box p \to \Diamond q)$

2. Show that the following rules are derivable in **K**:

   (a) If $\vdash_{\mathbf{K}} (\varphi_1 \wedge \varphi_2) \to \psi$, then $\vdash_{\mathbf{K}} (\Box\varphi_1 \wedge \Box\varphi_2) \to \Box\psi$.

   (b) If $\vdash_{\mathbf{K}} \varphi \to (\psi_1 \vee \psi_2)$, then $\vdash_{\mathbf{K}} \Diamond\varphi \to (\Diamond\psi_1 \vee \Diamond\psi_2)$.

3. Suppose Γ is a maximal **K**-consistent set. Prove the following:

   (a) If $\varphi \vee \psi \in \Gamma$, then either $\varphi \in \Gamma$ or $\psi \in \Gamma$.

   (b) If $\Box\varphi \in \Gamma$ and $\Box\psi \in \Gamma$, then $\Box(\varphi \wedge \psi) \in \Gamma$.