

SEMANTICS FOR PREDICATE LOGIC

PHIL 140A

SPRING 2016

1. Carry out the following substitutions:

- (a) $\exists x (P(x) \wedge \forall y (Q(y) \rightarrow R(x, y))) [y/x]$
- (b) $(\exists x R(x, c) \vee \forall y \neg P(c, x, y)) [c/x]$
- (c) $(\neg \forall y Q(f(x, y)) \wedge f(d, x) = z) [f(x, y)/x]$
- (d) $(\neg R(x, f(f(x))) \wedge \exists z (f(z) = f(x))) [z/f(x)][z/f(f(x))]$

2. Recall from Definition 3.4.4. the following definitions:

$$\begin{aligned} Cl(\varphi(z_1, \dots, z_n)) &:= \forall z_1 \cdots \forall z_n \varphi(z_1, \dots, z_n) \\ \mathfrak{A} \models \varphi &\Leftrightarrow \mathfrak{A} \models Cl(\varphi) \\ \models \varphi &\Leftrightarrow \text{for all } \mathfrak{A}, \mathfrak{A} \models \varphi \\ \mathfrak{A} \models \Gamma &\Leftrightarrow \text{for all } \varphi \in \Gamma, \mathfrak{A} \models \varphi \\ \Gamma \models \varphi &\Leftrightarrow \text{for all } \mathfrak{A}, \mathfrak{A} \models \Gamma \Rightarrow \mathfrak{A} \models \varphi. \end{aligned}$$

True or false:

- (a) $\{P(x)\} \models P(y)$.
 - (b) $\{P(x)\} \models \forall x P(x)$.
 - (c) For all formulas φ , and all models \mathfrak{A} , either $\mathfrak{A} \models \varphi$ or $\mathfrak{A} \models \neg \varphi$.
 - (d) For all formulas φ and ψ , if $\models \varphi \rightarrow \psi$, then $\{\varphi\} \models \psi$.
 - (e) For all formulas φ and ψ , if $\{\varphi\} \models \psi$, then $\models \varphi \rightarrow \psi$.
 - (f) For all formulas φ , $\models \varphi$ iff $\emptyset \models \varphi$.
3. Which of the following is true for all models \mathfrak{A} and all formulas φ and ψ ?
- (a) $\mathfrak{A} \models \neg \varphi$ iff $\mathfrak{A} \not\models \varphi$.
 - (b) $\mathfrak{A} \models \varphi \wedge \psi$ iff $\mathfrak{A} \models \varphi$ and $\mathfrak{A} \models \psi$.
 - (c) $\mathfrak{A} \models \varphi \vee \psi$ iff $\mathfrak{A} \models \varphi$ or $\mathfrak{A} \models \psi$.
 - (d) $\mathfrak{A} \models \varphi \rightarrow \psi$ iff $\mathfrak{A} \models \varphi$ only if $\mathfrak{A} \models \psi$.
 - (e) $\mathfrak{A} \models \varphi \leftrightarrow \psi$ iff $[\mathfrak{A} \models \varphi \text{ iff } \mathfrak{A} \models \psi]$.
 - (f) $\mathfrak{A} \models \forall x \varphi$ iff for all $a \in |\mathfrak{A}|$, $\mathfrak{A} \models \varphi[\bar{a}/x]$.
 - (g) $\mathfrak{A} \models \exists x \varphi$ iff for some $a \in |\mathfrak{A}|$, $\mathfrak{A} \models \varphi[\bar{a}/x]$.