Logic

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Basic Definitions

- The **premises** of an argument are the assumptions the argument invokes to derive its conclusion.
- An argument is **valid** if the conclusion is true in every situation in which all of the premises are true—in other words, it is impossible for all of the premises to be true and the conclusion false. If an argument is not valid, it is **invalid**.
- An argument is **sound** if (a) it is valid, and (b) every premise of the argument is true. If an argument is not sound, it is **unsound**.
- A **counterexample** to an argument is a possible situation that shows how the premises can all be true while the conclusion is false.
- A **basic premise** of an argument is a premise that is not derived from other premises in the argument. A **derived premise** is a premise that is derived from other premises in the argument.

Common Misconceptions

- *Arguments are not "true" or "false".* Sentences can be true or false. Arguments can only be valid/invalid, or sound/unsound.
- *Being valid does not mean having a true conclusion.* An argument is valid if the conclusion logically follows from the premises, i.e., *if* the premises are true, *then* the conclusion must be true.
- *To criticize an argument, it does not suffice to reject the conclusion.* One must also explain *why* an argument fails. There are only two possibilities: either the argument is invalid, or one of the premises of the argument is false. So, if an argument is valid, then the only way its conclusion is false is if one of the premises is false.
- *You do not need to refute every premise in order to refute the argument.* It's perfectly possible that an unsound argument has some true premises. Don't spread yourself thin by trying to refute every premise of an argument.

Examples

Valid and Sound

1. All dogs are mammals.
2. Old Yeller is a dog.

C. Old Yeller is a mammal.

Valid but Unsound

1. If the gods all love something, then it is just.
2. The gods all love pingpong.

C. Pingpong is just.

Valid and Unsound, but with True Conclusion

1. If the moon is made of cheese, then the moon is white.
2. The moon is not white.

C. The moon is not made of cheese.

Invalid (and hence Unsound)

1. If my bike was stolen, then my bike is fancy.
2. My bike is fancy.

C. My bike was stolen.

Invalid with True Conclusion

1. Everyone is rich or poor.
2. Donald Trump is rich.

C. Arc is poor.

Not all arguments are valid in virtue of their logical form. Common examples involve conceptually necessary entailments:

Valid, but Not in Virtue of Its Form

1. Strawberries are red.

C. Strawberries are colored.

Tips for Checking Validity

Most valid arguments you see in philosophy are valid in virtue of their *form*. For instance, consider the argument:

1. If the student did the reading, then they will pass.
2. The student did the reading.

C. Therefore, the student will pass.

This argument is valid. Its validity has nothing to do with students or readings or tests, though. It is due *entirely* to the form of the argument. We can read off the form more easily if we systematically replace "the student did the reading" and "the student will pass" with letters:

1. If A, then B.
2. A.

C. Therefore, B.

It does not matter what A and B stand for here: no matter what you plug in for A and B, it is impossible for premises 1 and 2 to be true while the conclusion C is false. Contrast the above argument with this one:

1. If the student did the reading, then they will pass.
2. The student will pass.

C. Therefore, the student did the reading.

The logical form of this argument is something like the following:

1. If A, then B.
2. B.

C. Therefore, A.

This argument is not valid. One way to see this is to construct a counterexample. For example, if the student got lucky and did not do the reading but passed anyway, the premises would be true while the conclusion would be false. Another way to that this argument is invalid is to plug in other sentences for A and B. For instance, if we let A stand for "oranges are blue" and let B stand for "oranges have a color", then the premises are clearly true while the conclusion is clearly false. Note, however, that this technique can only show that an argument is not valid *in virtue of its form*.

Common Valid Argument Forms

Modus Ponens

1. If A, then B.
2. A.

C. Therefore, B.

Modus Tollens

1. If A, then B.
2. Not B.

C. Therefore, not A.

Hypothetical Syllogism

1. If A, then B.
2. If B, then C.

C. Therefore, if A, then C.

Disjunctive Syllogism

1. Either A or B.
2. Not A.

C. Therefore, B.

Proof by Cases

1. Either A or B.
2. If A, then C.
3. If B, then C.

C. Therefore, C.

Reductio ad Absurdum

1. If A, then B.
2. If A, then also not B.

C. Therefore, not A.

Universal Instantiation

1. All Fs are Gs.
2. Alex is an F.

C. Therefore, Alex is a G.

Universal Transitivity

1. All Fs are Gs.
2. All Gs are Hs

C. Therefore, all Fs are Hs.

Other Tips

- If the conclusion of your argument reconstruction uses terms that do not appear in your premises, you probably messed up. Most (good) arguments in philosophy don't have this feature.
- If your argument reconstruction contains 7+ basic premises, you probably messed up. Most arguments you will come across in your philosophy courses are not *that* complicated.
- The only way to object to a valid argument is to reject one of the basic premises. You cannot simply reject the conclusion or a derived premise, since these follow from the basic premises.

Formal Logic Notation

Name	Symbol(s)	Meaning
Negation	$\neg, \sim, -, \bar{}$	"not"
Conjunction	$\wedge, \&, .$	"and"
Disjunction	$\vee$	"or"
(Material) Conditional	$\rightarrow, \supset$	"if. . . , then. . . "
(Material) Biconditional	$\leftrightarrow, \equiv$	"if and only if"
Universal Quantifier	$\forall x$	"for all x"
		"every x is such that"
Existential Quantifier	$\exists x$	"for some x"
		"there exists an x such that"

Examples of Notation

Throughout, let's use the following "symbol key":

a:	Ann
b:	Ben
$F(x)$:	x is friendly
$G(x)$:	x is gullible
$R(x,y)$:	x respects y

- $F(a) \rightsquigarrow$ Ann is friendly
- $\neg F(a) \rightsquigarrow$ Ann is not friendly
- $F(a) \wedge F(b) \rightsquigarrow$ Both Ann and Ben are friendly
- $F(b) \vee G(b) \rightsquigarrow$ Either Ben is friendly or he's gullible
- $G(a) \rightarrow G(b) \rightsquigarrow$ If Ann is gullible, then Ben is gullible
- $\forall x(G(x) \rightarrow F(x)) \rightsquigarrow$ Everyone who is gullible is friendly (more literally: for all x, if x is gullible, then x is friendly)
- $\exists x(G(x) \wedge F(x)) \rightsquigarrow$ Someone is gullible and friendly (more literally: for some x, x is gullible and x is friendly)
- $\neg \exists x(R(a,x) \wedge G(x)) \rightsquigarrow$ No one whom Ann respects is gullible
- $\forall x \exists y R(x,y) \rightsquigarrow$ Everyone respects someone (or other)
- $\exists y \forall x R(x,y) \rightsquigarrow$ Someone is respected by everyone

Inclusive vs. Exclusive Disjunction

Consider the following:

Either Ann went to the party or Ben went to the party.

Question: what if *both* Ann *and* Ben went to the party? In that case, is this sentence true?

- inclusive disjunction: yes ("either A or B (or both)")
- exclusive disjunction: no ("either A or B (but not both)")

In general, "$\vee$" refers to *inclusive* disjunction. Exclusive disjunction is often denoted with "$\veebar$".

- $A \vee B \rightsquigarrow$ Either A or B (or both)
- $A \veebar B \rightsquigarrow$ Either A or B (but not both)

Scope

Consider the following:

$$\neg F(a) \wedge F(b)$$

There are two ways to interpret this string of symbols:

- $\neg(F(a) \wedge F(b)) \rightsquigarrow$ It's not the case that both Ann and Ben are friendly (i.e., at least one of them is not friendly)
- $(\neg F(a) \wedge F(b)) \rightsquigarrow$ Ann is not friendly but Ben is friendly

Moral: using parentheses can help disambiguate otherwise ambiguous sentences.

Relative Quantifiers

English	Symbolization
All Fs are Gs	$\forall x(F(x) \rightarrow G(x))$
Some Fs are Gs	$\exists x(F(x) \wedge G(x))$
No Fs are Gs	$\neg \exists x(F(x) \wedge G(x))$
Only Fs are Gs	$\forall x(G(x) \rightarrow F(x))$