Proofs in LaTeX

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In what follows, I will demonstrate how to write natural deduction proofs in LaTeX from a variety of proof systems. The material of each (sub)section is (hopefully) organized by usefulness. In §2, we'll examine a couple of different methods for typing Fitch proofs. In §3, we'll see how to type proof trees (e.g., as in sequent calculi). In §4, a simple method for typing Lemmon proofs is demonstrated. In §5, we examine a couple of different methods for typing truth trees.

Contents

§1 Locating New `.sty` Files

In order to type certain natural deduction proofs in LaTeX, you may need to install some new `.sty` files that aren't preinstalled in your TeX distribution. There are two ways to do this: locally and globally.

The local way is to just place a copy of the `.sty` file in the same folder as your main `.tex` file. This is fine as far as it goes, but it gets annoying to have to include a different copy of this file each time you start a new `.tex` file.

The global way is to place just one copy of the `.sty` file in a globally accessible location, so that any `.tex` file you create has the ability to load that package automatically. The folder where you want to place the `.sty` file will vary based on your operating system.

Windows: `C:\Users\`⟨*user name*⟩`\texmf\tex\latex\local\`

Mac: `/Users/`⟨*user name*⟩`/Library/texmf/tex/latex/local/`

Linux: `~/texmf/tex/latex/local/`

§2 Fitch Proofs

There are a variety of ways to type Fitch proofs in LaTeX. Here, I will demonstrate the two most common.

§2.1 `fitch`

While a bit old (2003), `fitch` remains my favorite Fitch package. It's easy to use, and yet quite versatile.[1] I've placed a copy of `fitch.sty` here.

[1] A while back, I found that, for some reason, `fitch` conflicted with matrices. After some testing, I haven't been able to reproduce the problem. But just be aware of that…

§2.1.1 Basics

Fitch proofs are typed within the `fitch` environment:

```
\begin{fitch}
...
\end{fitch}
```

By default, the `fitch` will automatically number each line. To remove line numbers, use `fitch*` instead:

```
\begin{fitch*}
...
\end{fitch*}
```

As for typing the proofs, there are three essential commands:

- `\fa`: normal line
- `\fh`: hypothesis line
- `\fj`: first hypothesis line

Normal lines just have a vertical bar next to them. Hypothesis lines also have a horizontal line underneath them.

You can increase the nesting depth by iterating these commands. So, `\fa` makes a derived line in the main proof, `\fa \fa` makes a derived line in a subproof, `\fa \fa \fa` makes a derived line in a subsubproof, etc. Each line should end with `\\` like they do in tables. Everything is automatically in math mode.

Example 2.1: *Basic Fitch Proof*

1 | A
2 | | B
3 | | A
4 | $B \rightarrow A$
5 $A \rightarrow (B \rightarrow A)$

```
\begin{fitch}
  \fj  A \\
  \fa \fh B \\
  \fa \fa A \\
  \fa  B \rightarrow A \\
A \rightarrow (B \rightarrow A) \\
\end{fitch}
```

For spacing purposes, `\fj` is used on the last main premise line. Otherwise, use `\fh` for the hypothesis lines.

Example 2.2: *Failing to Use* `\fj`

1 | A
2 | $A \to B$
3 | B

```
\begin{fitch}
\fa A \\
\fh A \rightarrow B \\ % Ewww
\fa B
\end{fitch}
```

1 | A
2 | $A \to B$
3 | B

```
\begin{fitch}
\fa A \\
\fj A \rightarrow B \\ % Nice!
\fa B
\end{fitch}
```

§2.1.2 Rules

You can cite rules in a separate, nicely aligned column with & like in a table. Everything to the right of & is in text mode.

Example 2.3: *Rules*

1 | $A \wedge \neg A$
2 | A | $\wedge$-Elim (1)
3 | $\neg A$ | $\wedge$-Elim (1)
4 | $A \vee C$ | $\vee$-Intro (2)
5 | C | $\vee$-Elim (3, 4)

```
\begin{fitch}
\fj A \wedge \neg A  &  \\
\fa A                &  $\wedge$-Elim (1) \\
\fa \neg A           &  $\wedge$-Elim (1) \\
\fa A \vee C         &  $\vee$-Intro (2) \\
\fa C                &  $\vee$-Elim (3, 4) \\
\end{fitch}
```

§2.1.3 Boxes

You can create boxes around the contents of a line (as needed for quantifier proofs) using `\fw{`⟨*boxed item*⟩`}`.

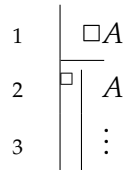
Example 2.4: *Using* `\fw`

1 | $\forall x Fx$
2 | | $\boxed{c}$
3 | | Fc
4 | | $Fc \vee Gc$
5 | $\forall x(Fx \vee Gx)$

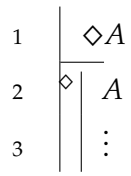
```
\begin{fitch}
\fj \forall x Fx \\
\fa \fh \fw{c} \\
\fa \fa Fc \\
\fa \fa Fc \vee Gc \\
\fa \forall x (Fx \vee Gx) \\
\end{fitch}
```

One can also use `\fn` for box modal proofs and `\fp` for diamond modal proofs:

Example 2.5: *Modal Proofs with* `\fn` *and* `\fn`




```
\begin{fitch}
\fj \Box A \\
\fa \fn A \\
\fa \fa \vdots \\
\end{fitch}
```

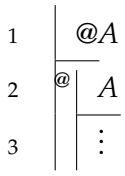



```
\begin{fitch}
\fj \Diamond A \\
\fa \fp A \\
\fa \fa \vdots \\
\end{fitch}
```

If you want to insert your own operator, you can use:

`\fitchmodal{`⟨*operator*⟩`}`
`\fitchmodalh{`⟨*operator*⟩`}` (for hypotheses)

Example 2.6: *Using* `\fitchmodalh`

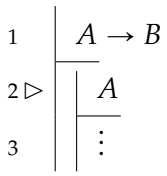



```
\begin{fitch}
\fj @A \\
\fa \fitchmodalh{@} A \\
\fa \fa \vdots \\
\end{fitch}
```

§2.1.4 Markers

You can put a marker in the proof to draw attention to a particular line using `\fr` (or `\fs`, if you don't want to include a vertical line).

Example 2.7: *Using* `\fr`




```
\begin{fitch}
\fj A \rightarrow B \\
\fr \fh A \\
\fa \fa \vdots \\
\end{fitch}
```

You can include multiple markers if you wish. To keep the marker from overlapping the vertical line, `\fr` should come before the other line commands.

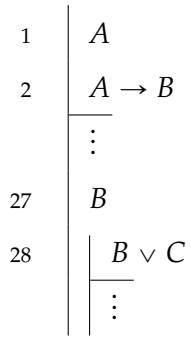
§2.1.5 Custom Tags/Skipping Numbers

If you want to modify what appears to the left of the outermost vertical line, you can do this manually using:

`\ftag{`⟨*lefthand side*⟩`}{`⟨*righthand side*⟩`}`

This is useful, e.g., for changing/removing line numbers from a proof.

Example 2.8: *Changing/Removing Line Numbers*

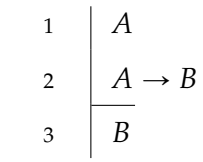



```
\begin{fitch}
\fa A \\
\fj A \rightarrow B \\
\ftag{~}{\fa \vdots} \\
\ftag{\scriptsize 27}{\fa B} \\
\ftag{\scriptsize 28}{\fa \fh B \vee C} \\
\ftag{~}{\fa \fa \vdots} \\
\end{fitch}
```

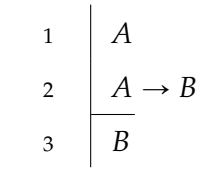
§2.1.6 Shorter Top Line

If you think the top of the main vertical line extends too far, you can use `\fb` instead of `\fa` there. (It's hard to tell the difference.)

Example 2.9: *Comparing Vertical Line Length with* `\fb`




```
\begin{fitch}
\fb A \\ % Right length
\fj A \then B \\
\fa B
\end{fitch}
```




```
\begin{fitch}
\fa A \\ % A bit too long
\fj A \then B \\
\fa B
\end{fitch}
```

§2.1.7 An Illustrative Example

Example 2.10: *Illustrating* `fitch`

1	$\forall x(Fx \to Gx)$	
2	$\exists x Fx$	
3	$\boxed{c} : Fc$	
4	$Fc \to Gc$	$\forall$-Elim (1)
5	Gc	$\to$-Elim (3, 4)
6	$\exists x Gx$	$\exists$-Intro (5)
7	$\exists x Gx$	$\exists$-Elim (2, 3–6)
8	$\exists x Fx \to \exists x Gx$	$\to$-Intro (2–7)
	$\triangleright\ \forall x(Fx \to Gx) \to (\exists x Fx \to \exists x Gx)$	$\to$-Intro (1–8)

```
\begin{fitch}
\fj \forall x (Fx \rightarrow Gx)  &  \\ %1
\fa \fh \exists x Fx               &  \\ %2
\fa \fa \fh \fw{c}: Fc             &  \\ %3
\fa \fa \fa Fc \rightarrow Gc      &  $\forall$-Elim (1) \\ %4
\fa \fa \fa Gc                     &  $\rightarrow$-Elim (3, 4) \\ %5
\fa \fa \fa \exists x Gx           &  $\exists$-Intro (5) \\ %6
\fa \fa \exists x Gx               &  $\exists$-Elim (2, 3--6) \\ %7
\fa \exists x Fx \rightarrow \exists x Gx  &  $\rightarrow$
-Intro (2--7) \\ %8
\ftag{~}{\fs \forall x (Fx \rightarrow Gx) \rightarrow (\exists x Fx
\rightarrow \exists x Gx)}  &  $\rightarrow$-Intro (1--8) \\ %End
\end{fitch}
```

§2.2 `lplfitch`

If you used *Language, Proof, and Logic* as your introductory logic text, you may be happy to hear that you can typeset Fitch proofs just like that book. I find `fitch` easier to read and use, but you may find `lplfitch`'s organizational structure more suitable for your purposes.

Installing `lplfitch.sty` is a bit roundabout. So to make your life easy, I've just placed the most up-to-date version as of January 2017 here. To download the most recent version, if it has changed, you need to first go to its CTAN page and download the `.zip` archive. You then need to run `lplfitch.ins` (just typeset as normal), and it will generate the `lplfitch.sty` file.

§2.2.1 Basics

Instead of placing your fitch proof in an environment, you simply place it inside the command:

```
\fitchprf{⟨hypotheses⟩}{⟨deduction⟩}
```

The first argument is a list of your hypotheses, while the second argument is your deduction.

To add a line in a deduction, use the command:

```
\pline[⟨left of formula⟩]{⟨formula⟩}[⟨rule⟩]
```

You can separate lines using \\. (In the examples that follow, I'll be generous with spacing to make it easier to read. But you do not have to be so liberal.)

Example 2.11: *Basic Fitch Proof*

A
A → B

B

```
\fitchprf{
  \pline{A} \\
  \pline{A \rightarrow B} }
{ \pline{B} }
```

To start a subproof, all you need to do is use the command:

```
\subproof{⟨hypotheses⟩}{⟨deduction⟩}
```

which works exactly like `\fitchprf`. You do not need to put \\ at the end of `\subproof` to move on to the next line.

Example 2.12: *Subproofs*

A

| B
| ---
| A

B → A

```
\fitchprf{
  \pline{A} }
{ \subproof{\pline{B}}
  { \pline{A} }
  \pline{B \rightarrow A} }
```

§2.2.2 Line Numbers

Unlike `fitch.sty`, there's no good way to automatically number lines. You can do this manually via the optional argument for `\pline`.

Example 2.13: *Numbered Lines*

1. A

| 2. B
| ---
| 3. A

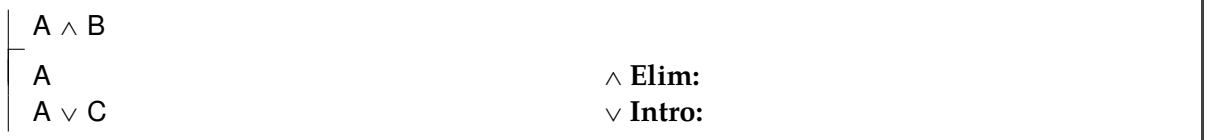
4. B → A

```
\fitchprf{
  \pline[1.]{A} }
{ \subproof{\pline[2.]{B}}
  { \pline[3.]{A} }
  \pline[4.]{B \rightarrow A} }
```

§2.2.3 Rules

The `\pline` command's second optional argument allows you to cite rules in your proofs.

Example 2.14: *Rules*




```
\fitchprf{
  \pline{A \wedge B} }
{ \pline{A}[$\wedge$-Elim] \\
  \pline{A \vee C}[$\vee$-Intro] }
```

As you can see, for some reason, the formulas are placed ridiculously far away from their rule citations. The easiest fix for me has been to manually reset the width of fitch proofs by adding the following line of code before the proof:

```
\setlength{\fitchprfwidth}{⟨length⟩}
```

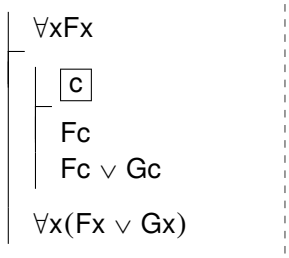
§2.2.4 Boxes

For quantifier proofs, boxed subproofs may be added using the command:

```
\boxedsubproof[⟨left of boxed item⟩]{⟨boxed item⟩}{⟨hypotheses⟩}{⟨deduction⟩}
```

This works exactly like `\fitchprf` and `\subproof`. The optional argument allows you to add the line number to the left of the box, instead of to the left of the formula. (Note: don't use `\pline` in the ⟨*hypothesis*⟩ argument.)

Example 2.15: *Boxed Subproof*




```
\fitchprf{
  \pline{\forall x Fx}
}
{ \boxedsubproof{c}{}
  { \pline{Fc} \\
    \pline{Fc \vee Gc} }
  \pline{\forall x (Fx \vee Gx)} }
```

§2.2.5 Symbols

`lplfitch` provides a small library of predefined commands for common logic symbols. Below is a table of these commands.

Command	Output
\lnot	¬
\land	∧
\lor	∨
\lif	→
\liff	↔
\lfalse	⊥
\lall	∀
\lis	∃
\uni{x}	$\forall x$
\exi{x}	$\exists x$

Command	Output
\lnoti{1}	¬ **Intro:** 1
\lnote{2}	¬ **Elim:** 2
\landi{3}	∧ **Intro:** 3
\lande{4}	∧ **Elim:** 4
\lori{5}	∨ **Intro:** 5
\lore{6}{7}{8}	∨ **Elim:** 6, 7, 8
\lifi{9}	→ **Intro:** 9
\life{10}	→ **Elim:** 10,
\liffi{11}	↔ **Intro:** 11,
\liffe{12}	↔ **Elim:** 12,
\lfalsei{13}	⊥ **Intro:** 13,
\lfalsee{14}	⊥ **Elim:** 14
\lalli{15}	∀ **Intro:** 15
\lalle{16}	∀ **Elim:** 16
\lexii{17}	∃ **Intro:** 17
\lexie{18}{19}	∃ **Elim:** 18, 19
\reit{20}	**Reit:** 20
\eqi	= **Intro**
\eqe{21}{22}	= **Elim:** 21, 22

Table 1: Symbol commands in `lplfitch`.

The difference between `\lall` and `\uni` is in spacing. Compare:

`\lall x Fx`	⇒	$\forall x Fx$	`\lall x \lis y Rxy`	⇒	$\forall x \exists y Rxy$
`\uni{x} Fx`	⇒	$\forall x\ Fx$	`\uni{x} \exi{y} Rxy`	⇒	$\forall x\ \exists y\ Rxy$

`lplfitch` also introduces a `\formula{`⟨*formula*⟩`}` command, so that you can indicate what is and is not a formula. By default, `\formula` puts its argument into sans serif. Everything inside `\pline` is inside the scope of `\formula`. You can change the formatting however you like by redefining the `\formula` command. For instance, to just have everything appear in normal math mode font, you can insert the following line of code in the preamble:

```
\renewcommand{\formula}[1]{\ensuremath{#1}}
```

This is why the examples here don't appear in sans serif.

If one introduces ellipses, one should use `\ellipsesline` in place of `\pline{\vdots}` to get the spacing right. Compare:

`\vdots`

A
⋮
B

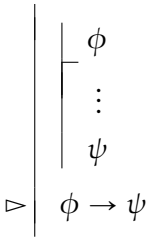
`\ellipsisline`

A
⋮
B

§2.2.6 `fitchctx`

`lplfitch` provides another command for making Fitch proofs in place of `\fitchprf`: `\fitchctx`. This command, among other things, allows you to remove the first horizontal hypothesis line. Unlike `\fitchprf`, `\fitchctx` only takes one argument.

Example 2.16: *Removing the First Hypothesis Line*

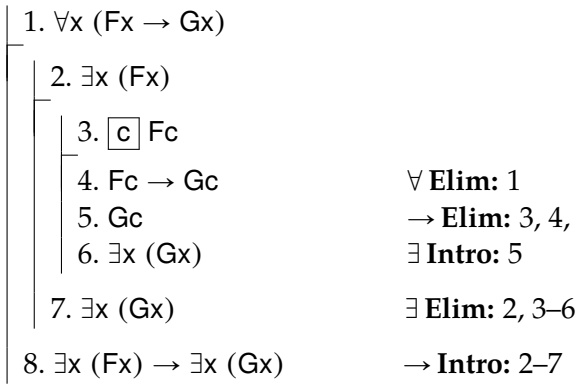



```
\fitchctx{
  \subproof{ \pline{\phi} }
  { \ellipsesline \\
    \pline{\psi} }
    \fpline{\phi \rightarrow \psi} }
```

Notice the last line's use of `\fpline` instead of `\pline`. This places the marker on that line. *Note that* `\fpline` *is only defined in* `fitchctx`. You can redefine the marker used by `\fpline` by redefining the command `\slider`.

§2.2.7 An Illustrative Example

Example 2.17: *Illustrating* `lplfitch`




```
\setlength{\fitchprfwidth}{5cm}

\fitchprf{ \pline[1.]{\uni{x} (Fx \lif Gx)} }
{ \subproof{ \pline[2.]{\exi{x} (Fx)} }
  { \boxedsubproof[3.]{c}{ Fc }
    { \pline[4.]{Fc \lif Gc}[\lalle{1}] \\
      \pline[5.]{Gc}[\life{3, 4}] \\
      \pline[6.]{\exi{x} (Gx)}[\lexii{5}] }
    \pline[7.]{\exi{x} (Gx)}[\lexie{2}{3--6}] }
  \pline[8.]{\exi{x} (Fx) \lif \exi{x} (Gx)}[\lifi{2--7}] }
```

§3 Proof Trees

When it comes to making proof trees, there's only one really good option: `bussproofs`. I've placed a copy of `bussproofs.sty` here. The official website with full documentation can be found here.

In what follows, we'll just go over the essentials. You can find a more complete documentation of `bussproofs` with more formatting tricks here.

§3.1 Basics

To make a proof tree, one starts with the `prooftree` environment.

```
\begin{prooftree}
...
\end{prooftree}
```

There are two ways to type proof trees in `bussproofs`: the *automatic* way and the *manual* way. We'll look at both.

§3.1.1 Automatic

Each node of a proof tree will be enclosed in one of the following commands (the `C` stands for "centered"):

- `\AxiomC{`⟨*node*⟩`}`: the beginning, or "leaf", of a tree
- `\UnaryInfC{`⟨*node*⟩`}`: a node that follows from 1 previous node
- `\BinaryInfC{`⟨*node*⟩`}`: a node that follows from 2 previous nodes
- `\TrinaryInfC{`⟨*node*⟩`}`: ...3 previous nodes
- `\QuaternaryInfC{`⟨*node*⟩`}`: ...4 previous nodes
- `\QuinaryInfC{`⟨*node*⟩`}`: ...5 previous nodes

The ⟨*node*⟩ is in text mode. The ordering of the commands determines their location on the tree. Inference lines are placed directly below the most recently created nodes. So for instance, suppose my first three nodes are:

```
\AxiomC{$A_1$}
\AxiomC{$A_2$}
\AxiomC{$A_3$}
```

If my next line were `\UnaryInfC{$B$}`, B would be placed directly below A_3. If my next line were `\BinaryInfC{$B}$`, B would be placed directly below A_2 and A_3. And so on. If I want to draw an inference after just A_2, say, then I need to write `\UnaryInfC{$B$}` after `\AxiomC{$A_2$}`. This rule applies to all nodes, including those derived via inferences.

Example 3.1: *Basic Proof Tree*

$$\dfrac{\dfrac{A_1}{A_2} \qquad \dfrac{B_1 \qquad B_2}{B_3}}{AB}$$

```
\begin{prooftree}
\AxiomC{$A_1$}
  \UnaryInfC{$A_2$}
\AxiomC{$B_1$}
\AxiomC{$B_2$}
  \BinaryInfC{$B_3$}
    \BinaryInfC{$AB$}
\end{prooftree}
```

§3.1.2 Manual

Nodes on the automatic way are centered. But often, you may want to align nodes based to a symbol, e.g., $\vdash$ or $\Rightarrow$. The manual way can help.

The commands are the same as before, except without the `C` or the curly brackets `{...}`. Furthermore, your arguments must be in math mode, and must include the command `\fCenter` so that `bussproofs` knows where to align the node. Compare the two methods:

Example 3.2: *Automatic vs. Manual*

$$\dfrac{A \vdash B}{A, A', A'' \vdash B}$$

```
\begin{prooftree}
\AxiomC{$A \vdash B$}
  \UnaryInfC{$A, A', A'' \vdash B$}
\end{prooftree}
```

$$\dfrac{A \vdash B}{A, A', A'' \vdash B}$$

```
\begin{prooftree}
\Axiom$A \fCenter\vdash B$
  \UnaryInf$A, A', A'' \fCenter\vdash B$
\end{prooftree}
```

By default, `\fCenter` just produces a space. To make your life easy, you could define it as a turnstile or an arrow. So for instance, if we insert this into the preamble:

```
\renewcommand{\fCenter}{\vdash}
```

then `\fCenter` will output $\vdash$; so the code above could be replaced by:

Example 3.3: *Redefined* `\fCenter`

$$\dfrac{A \vdash B}{A, A', A'' \vdash B}$$

```
\begin{prooftree}
\Axiom$A \fCenter B$
  \UnaryInf$A, A', A'' \fCenter B$
\end{prooftree}
```

You can use both manual and automatic methods in the same proof tree. So there's no reason to feel like you must stick with one method or the other.

§3.2 Rules and Lines

§3.2.1 Rules

You can add rule labels with `\LeftLabel{⟨label⟩}` and `\RightLabel{⟨label⟩}`. These should be placed before the inference line is drawn.

Example 3.4: *Right Label*

```
\begin{prooftree}
\AxiomC{$A \vdash C$}
  \RightLabel{Weakening}
  \UnaryInfC{$A, B \vdash C$}
\end{prooftree}
```

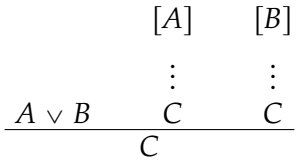
§3.2.2 Lines

You can also change the appearance of an inference line. On the one hand,

```
\noLine
\singleLine
\doubleLine
```

will draw zero, one, or two lines respectively for the next inference.

Example 3.5: *Drawing No Lines*



```
\begin{prooftree}
\AxiomC{$A \vel B$}
\AxiomC{$[A]$}
  \noLine
  \UnaryInfC{$\vdots$}
  \noLine
  \UnaryInfC{$C$}
\AxiomC{$[B]$}
  \noLine
  \UnaryInfC{$\vdots$}
  \noLine
  \UnaryInfC{$C$}
    \TrinaryInfC{$C$}
\end{prooftree}
```

Example 3.6: *Drawing Double Lines*

$$\dfrac{A, B \vdash C}{\dfrac{}{A \vdash B \to C}}$$

```
\begin{prooftree}
\AxiomC{$A, B \vdash C$}
  \doubleLine
  \UnaryInfC{$A \vdash B \rightarrow C$}
\end{prooftree}
```

On the other hand,

```
\solidLine
\dashedLine
\dottedLine
```

will draw solid, dashed, or dotted lines respectively for the next inference.

Example 3.7: *Drawing Dashed Line*

$$\dfrac{A \vdash C}{\dfrac{}{A, B \vdash C}}$$

```
\begin{prooftree}
\AxiomC{$A \vdash C$}
  \doubleLine
  \dashedLine
  \UnaryInfC{$A, B \vdash C$}
\end{prooftree}
```

As the above example illustrates, you can mix and match these style options.

If you want all of your inference lines to be a certain style, the following global commands will change the default style accordingly:

```
\alwaysNoLine
\alwaysSingleLine
\alwaysDoubleLine
\alwaysSolidLine
\alwaysDashedLine
\alwaysDottedLine
```

§3.3 Abbreviations

If your fingers get tired, you can utilize the abbreviated versions of these commands. Simply insert `\EnableBpAbbreviations` in your document. Then you can make use of the following abbreviated commands:

Abbrevation	Abbreviates	Abbrevation	Replaces
\AX	\Axiom	\AXC	\AxiomC
\UI	\UnaryInf	\UIC	\UnaryInfC
\BI	\BinaryInf	\BIC	\BinaryInfC
\TI	\TernaryInf	\TIC	\TernaryInfC

Table 2: Abbreviations provided by bussproofs.

I've also defined some abbreviated commands. If you want to use them, add these lines to your preamble (note this requires the xargs, ifthen, and xifthen packages):

```
\newcommand{\ife}[4]{\ifthenelse{\equal{#1}{#2}}{#3}{#4}}
\newenvironment{tree}{\begin{prooftree}}{\end{prooftree}}
\newcommand{\ax}[1]{\AxiomC{\EMX{#1}}}
\renewcommandx{\inf}[2][1=1, usedefault]{
\ife{#1}{1}
  {\UnaryInfC{\EMX{#2}}}
  {\ife{#1}{2}
    {\BinaryInfC{\EMX{#2}}}
    {\ife{#1}{3}
      {\TrinaryInfC{\EMX{#2}}}
      {\ife{#1}{4}
        {\QuaternaryInfC{\EMX{#2}}}
        {\ife{#1}{5}
          {\QuinaryInfC{\EMX{#2}}}
          {}
        }
      }
    }
  }
}
\newcommand{\LL}[1]{\LeftLabel{#1}}
\newcommand{\RL}[1]{\RightLabel{#1}}
\newcommand{\binf}[1]{\inf[2]{#1}}
```

Abbreviation	Abbreviates
tree	prooftree
\ax	\AxiomC
\inf	\UnaryInf
\binf	\BinaryInfC
\LL	\LeftLabel
\RL	\RightLabel

Table 3: Abbrevations provided by me.

The abbreviations already set their inputs in math mode (except for \LL and \RL), so you don't need to include and $$'s. The \inf[⟨*number*⟩]{⟨*label*⟩} command has an optional parameter, which determines the number of nodes above that inference line. The default option is 1. So:

```
\inf[1]  ⇒  \UnaryInfC
\inf[2]  ⇒  \BinaryInfC
\inf[3]  ⇒  \TernaryInfC
\inf[4]  ⇒  \QuaternaryInfC
\inf[5]  ⇒  \QuinaryInfC
\inf[?]  ⇒  nothing
```

§3.4 An Illustrative Example

Example 3.8: *Illustrating* bussproofs

$$\rightarrow\text{E}\,\dfrac{\dfrac{(A \rightarrow B) \wedge (A \rightarrow C) \vdash A \rightarrow B \qquad (A \rightarrow B) \wedge (A \rightarrow C) \vdash A \rightarrow C}{(A \rightarrow B) \wedge (A \rightarrow C), A \vdash B \qquad (A \rightarrow B) \wedge (A \rightarrow C), A \vdash C}\,\rightarrow\text{E}}{\dfrac{(A \rightarrow B) \wedge (A \rightarrow C), A \vdash B \wedge C}{(A \rightarrow B) \wedge (A \rightarrow C) \vdash A \rightarrow (B \wedge C)}\,\rightarrow\text{I}}\,\wedge\text{I}$$

```
\renewcommand{\fCenter}{\vdash}

\begin{prooftree}

% Left Column
\Axiom$(A \rightarrow B) \wedge (A \rightarrow C) \fCenter A \rightarrow B$
\LeftLabel{$\rightarrow$E}
  \UnaryInf$(A \rightarrow B) \wedge (A \rightarrow C), A \fCenter B$

% Right Column
\AxiomC{$(A \rightarrow B) \wedge (A \rightarrow C) \vdash A \rightarrow C$}
\RightLabel{$\rightarrow$E}
\dashedLine
  \UnaryInfC{$(A \rightarrow B) \wedge (A \rightarrow C), A \vdash C$}

% Combining Columns
\RightLabel{$\wedge$I}
\BinaryInfC{$(A \rightarrow B) \wedge (A \rightarrow C), A \vdash B \wedge C$}
\RightLabel{$\rightarrow$I}
\doubleLine
\UnaryInfC{$(A \rightarrow B) \wedge (A \rightarrow C) \vdash A \rightarrow
(B \wedge C)$}

\end{prooftree}
```

§4 Lemmon Proofs

The easiest way to type Lemmon proofs is with `ND`, though I typically end up drawing these proofs using TikZ (see §5.2). You can find a copy of `ND` here. A good, quick guide can be found here.

Typing Lemmon proofs are relatively straightforward. Lemmon proofs are typed inside the `ND` environment:

```
\begin{ND}[⟨title⟩][⟨label⟩][⟨premise width⟩][⟨rule width⟩][⟨total width⟩]
...
\end{ND}
```

The easiest way to control spacing is, I think, to set the total width first, and make adjustments from there if necessary.

Lines in a proof are made using the `\ndl` command:

```
\ndl{⟨premises⟩}{⟨formula⟩}{⟨rule⟩}
```

One can label lines of a proof (unlike in other proof systems we've seen) to automatically refer back to them when you cite rules. However, this feature seems to conflict with the `hyperref` package. So unfortunately, I can't typeset a proof with labels here. Again, see this guide for details. Otherwise, using `ND` is pretty straightforward:

Example 4.1: *Illustrating* `ND`

Lemmon Style

1	(1)	A	Premise
2	(2)	$A \rightarrow B$	Premise
3	(3)	$B \rightarrow C$	Premise
1, 2	(4)	B	→-Elim (1, 2)
1, 2, 3	(5)	C	→-Elim (3, 4)

```
\begin{ND}[Lemmon Style][][][][0.7\linewidth]

\ndl{1}{$A$}{Premise}
\ndl{2}{$A \rightarrow B$}{Premise}
\ndl{3}{$B \rightarrow C$}{Premise}
\ndl{1,2}{$B$}{$\rightarrow$-Elim (1, 2)}
\ndl{1,2,3}{$C$}{$\rightarrow$-Elim (3, 4)}

\end{ND}
```

§5 Truth Trees

I only know of two ways to type truth trees in LaTeX. The first is to use `qtree`, which is simple and easy to learn, but has very low functionality, and is more designed for constructing parsing trees than truth trees. In particular, I haven't found a good way to label or number nodes of the tree in a separate column, and customizing the appearance of the tree is unnatural. The second way is to use TikZ, which is much more flexible and still fairly straightforward to use.

§5.1 `qtree`

I've placed a copy of the `qtree` package here. The official site is here.

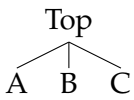
§5.1.1 Basics

To type a tree, you simply use the command:

```
\Tree[.{⟨parent⟩} {⟨child1⟩} {⟨child2⟩} ... ]
```

Each child is separated by a space.

Example 5.1: *Basic Tree*

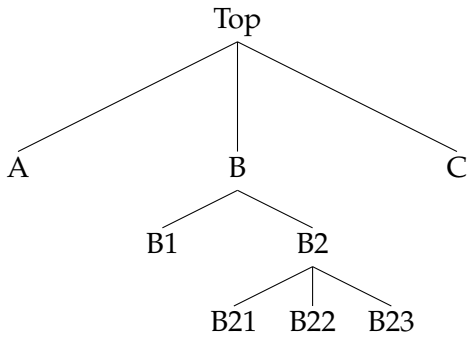



```
\Tree[.{Top} {A} {B} {C} ]
```

The general pattern for including more descendent nodes is to enclose the nodes in square brackets and to place a period in front of the parent node. You may include the period and parent node either after the left or right square bracket. Thus, either of these patterns is acceptable, and produces the same output:

```
[.{⟨parent⟩} {⟨child1⟩} {⟨child2⟩} ... ]
[ {⟨child1⟩} {⟨child2⟩} ... ].{⟨parent⟩}
```

Example 5.2: *Further Branching*



```
\Tree[.{Top} {A} [.{B} {B1} [.{B2} {B21} {B22} {B23} ] ] {C} ]
```

```
\Tree[ {A} [ {B1} [ {B21} {B22} {B23} ].{B2} ].{B} {C} ].{Top}
```

§5.1.2 Alignment

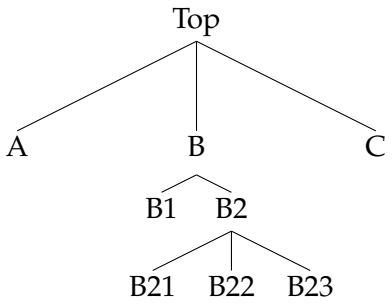
By default, trees are always centered on the page. You can change this globally by adding the option `nocenter` when you load the package. You can also toggle automatic centering locally via the following commands:

```
\qtreecentertrue
\qtreecenterfalse
```

§5.1.3 Spacing

To adjust the inter-node distance, you can use the command `\qsetw{`⟨*width*⟩`}`. This adjusts the width of (sub)tree whose last node appears to the left of the command (do not forget the `!` before `\qsetw`).

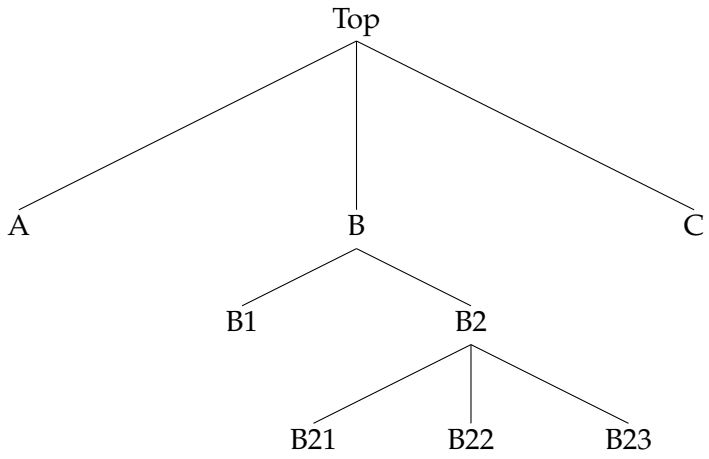
Example 5.3: *Adjusting Node Distance*




```
\Tree[.{Top} {A} [.{B} {B1} [.{B2} {B21} {B22} {B23} ] !\qsetw{1cm} ] {C} ]
```

Notice also that the subtrees gradually get smaller. To avoid this, you can use `\qbalance`.

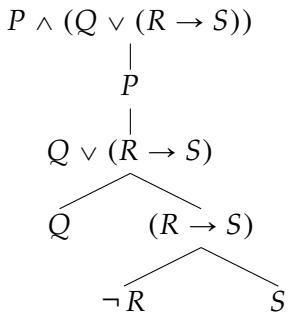
Example 5.4: *Balancing Tree*




```
\Tree[.{Top} {A} [.{B} {B1} [.{B2} {B21} {B22} {B23} !{\qbalance} ]  ] {C} ]
```

§5.1.4 An Illustrative Example

Example 5.5: *Illustrating* `qtree`




```
\Tree[.{$P \wedge (Q \vee (R \rightarrow S))$} [.{$P$} [.{$Q \vee (R
\rightarrow S)$} {$Q$} [.{$(R \rightarrow S)$} {$\neg R$} {$S$} !{\qbalance} ]
!\qsetw{3cm} ] ] ]
```

§5.2 TikZ

While TikZ can be a bit unweildly to learn in its full scope (see the 1161-page manual!), using it for simple things like truth trees is rather easy and flexible once things are set up. In what follows, we won't try to give a general tutorial for TikZ; we'll only set things up just enough to do truth trees. See my guide to TikZ here for more on how to use TikZ to create diagrams in LaTeX. For a more complete tutorial, I would recommend reading the first few chapters of the manual, which is quite helpful in walking you through examples step-by-step.

§5.2.1 Set-up

Include the following in your preamble:

```
\usepackage{tikz}
\usetikzlibrary{positioning}
```

While TikZ offers a wide variety of other libraries which may help customize all sorts of things, this suffices for truth trees.

§5.2.2 Nodes

We'll show how to draw a complete proof tree one step at a time. The first thing to do is to figure out how to draw the nodes of the truth tree.

Start with the following:

```
\begin{tikzpicture}[node distance=1ex]
...
\end{tikzpicture}
```

The optional parameter isn't necessary, and you can adjust the distance however you like, though 1ex seems appropriate.

A line of code corresponding to a node will look generally like this:

```
\node (⟨name⟩) [⟨options⟩] {⟨formula⟩};
```

Don't forget the semicolon at the end! You'll get errors if you do (I make this mistake a lot…).

The ⟨*name*⟩ corresponds to a unique name for that node (no spaces). Usually, for ease of reference, I try to have it mimic the formula. For instance, if $A \to B$ is your formula, you could have its name be `A->B`, or `AtoB`.

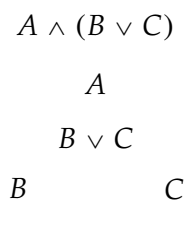
The ⟨*options*⟩ will help position the formula appropriately relative to the other formulas. The most common options for truth trees include:

```
below=of ⟨name⟩
below left=of ⟨name⟩
below right=of ⟨name⟩
```

The ⟨*formula*⟩ is where you can write your formula. You can also just put ordinary text there. Whatever you fill in for this argument will go in the center of the node.

In the examples below, I put spaces between these different arguments for readability. But they are not necessary.

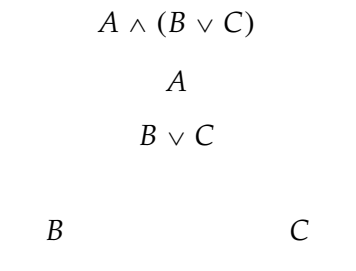
Example 5.6: *Drawing Nodes*




```
\begin{tikzpicture}[node distance=1ex]
  \node  (A&BvC)                      {$A \wedge (B \vee C)$};
  \node  (A)     [below=of A&BvC]     {$A$};
  \node  (BvC)   [below=of A]         {$B \vee C$};
  \node  (B)     [below left=of BvC]  {$B$};
  \node  (C)     [below right=of BvC] {$C$};
\end{tikzpicture}
```

If you want to adjust the spacing between nodes, you can put a length before `of` in the optional parameters.

Example 5.7: *Adjusting Length*




```
\begin{tikzpicture}[node distance=1ex]
  \node  (A&BvC)                         {$A \wedge (B \vee C)$};
  \node  (A)     [below=of A&BvC]        {$A$};
  \node  (BvC)   [below=of A]            {$B \vee C$};
  \node  (B)     [below left=1cm of BvC] {$B$};
  \node  (C)     [below right=1cm of BvC] {$C$};
\end{tikzpicture}
```

If you want to manually shift the position of a node, you can do so via `xshift` and `yshift`.

Example 5.8: *Adjusting Position*

$$A \wedge (B \vee C)$$

$$A$$

$$B \vee C$$

$$B \qquad C$$

```
\begin{tikzpicture}[node distance=1ex]
  \node  (A&BvC)                                  {$A \wedge (B \vee C)$};
  \node  (A)    [below=of A&BvC]                  {$A$};
  \node  (BvC)  [below=of A]                      {$B \vee C$};
  \node  (B)    [below left=of BvC, xshift=5mm]   {$B$};
  \node  (C)    [below right=of BvC, xshift=-5mm] {$C$};
\end{tikzpicture}
```

§5.2.3 Paths

To draw lines connecting formulae, insert a line of code like the following:

```
\path  (⟨name of start node⟩)  edge[-]  (⟨name of end node⟩);
```

Example 5.9: *Drawing Paths*

$$A \wedge (B \vee C)$$

$$A$$

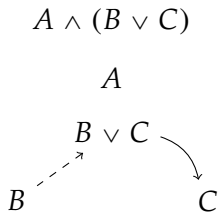
$$B \vee C$$

$$B \qquad\qquad C$$

```
\begin{tikzpicture}[node distance=1ex]
  \node  (A&BvC)                          {$A \wedge (B \vee C)$};
  \node  (A)     [below=of A&BvC]         {$A$};
  \node  (BvC)   [below=of A]             {$B \vee C$};
  \node  (B)     [below left=5mm of BvC]  {$B$};
  \node  (C)     [below right=5mm of BvC] {$C$};

  \path  (BvC)  edge[-]  (B);
  \path  (BvC)  edge[-]  (C);
\end{tikzpicture}
```

Next to the -, you can add optional arguments to change the shape or bend of the line. For instance, you can make it `dashed` or `dotted`, or you could `bend right` or `bend left`. You can also change - to an arrow ->.

Example 5.10: *Shaping Paths*

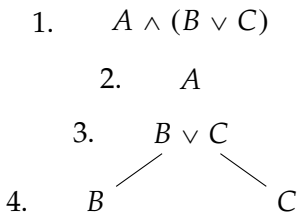



```
\begin{tikzpicture}[node distance=1ex]
  \node  (A&BvC)                          {$A \wedge (B \vee C)$};
  \node  (A)    [below=of A&BvC]          {$A$};
  \node  (BvC)  [below=of A]              {$B \vee C$};
  \node  (B)    [below left=5mm of BvC]   {$B$};
  \node  (C)    [below right=5mm of BvC] {$C$};

  \path  (BvC)  edge[<-, dashed]  (B);
  \path  (BvC)  edge[->, bend left=45]  (C); % Bend in degrees
\end{tikzpicture}
\end{tikzpicture}
```

§5.2.4 Numbers

Line numbers can themselves be treated as nodes.

Example 5.11: *Line Numbers*




```
\begin{tikzpicture}[node distance=1ex]
  \node  (A&BvC)                          {$A \wedge (B \vee C)$};
  \node  (A)    [below=of A&BvC]          {$A$};
  \node  (BvC)  [below=of A]              {$B \vee C$};
  \node  (B)    [below left=5mm of BvC]   {$B$};
  \node  (C)    [below right=5mm of BvC] {$C$};
```

```
  \path  (BvC)  edge[-]  (B);
  \path  (BvC)  edge[-]  (C);

  \node  (1)  [left=5mm of A&BvC]  {1.};
  \node  (2)  [left=5mm of A]      {2.};
  \node  (3)  [left=5mm of BvC]    {3.};
  \node  (4)  [left=5mm of B]      {4.};
\end{tikzpicture}
```

However, that doesn't look very nice. To align the numbers vertically, we need to consider an expansion of the `\node` command:

```
\node  (⟨name⟩)  at (⟨position⟩)  [⟨options⟩]  {⟨formula⟩};
```

A lot of things could be specified with ⟨*position*⟩. But one in particular makes use of `|-` and `-|`. If one writes before the optional parameters

```
at (⟨name_v⟩ |- ⟨name_h⟩)
```

then the resulting node will be *vertically* aligned to ⟨$name_v$⟩ and *horizontally* aligned to ⟨$name_h$⟩. The result is the same if one writes instead:

```
at (⟨name_h⟩ -| ⟨name_v⟩)
```

Example 5.12: *Line Numbers Aligned*

1. $A \wedge (B \vee C)$
2. A
3. $B \vee C$
4. B C

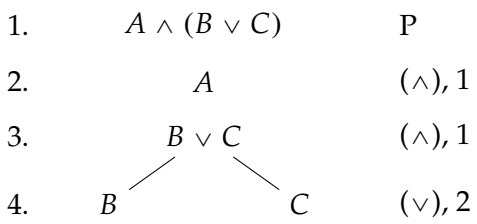
```
\begin{tikzpicture}[node distance=1ex]
  \node  (A&BvC)                          {$A \wedge (B \vee C)$};
  \node  (A)     [below=of A&BvC]         {$A$};
  \node  (BvC)   [below=of A]             {$B \vee C$};
  \node  (B)     [below left=5mm of BvC]  {$B$};
  \node  (C)     [below right=5mm of BvC] {$C$};

  \path  (BvC)  edge[-]  (B);
  \path  (BvC)  edge[-]  (C);

  \node  (1)  [left=1cm of A&BvC]  {1.};
```

```
  \node  (2)  at (1 |- A)          {2.};
  \node  (3)  at (1 |- BvC)        {3.};
  \node  (4)  at (1 |- B)          {4.};
\end{tikzpicture}
```

§5.2.5 Rules

Rule citations can be done in the same way that numbers are done. However, to ensure that the rule citations are aligned properly, I would recommend putting the label in the optional parameters rather than as the formula.

Example 5.13: *Line Numbers Aligned*

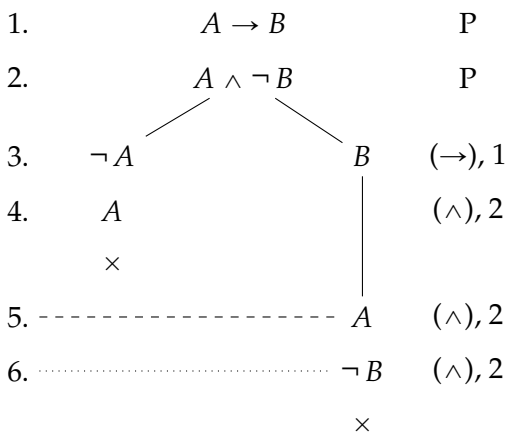



```
\begin{tikzpicture}[node distance=1ex]
  \node  (A&BvC)                          {$A \wedge (B \vee C)$};
  \node  (A)    [below=of A&BvC]          {$A$};
  \node  (BvC)  [below=of A]              {$B \vee C$};
  \node  (B)    [below left=5mm of BvC]   {$B$};
  \node  (C)    [below right=5mm of BvC]  {$C$};

  \path  (BvC)  edge[-]  (B);
  \path  (BvC)  edge[-]  (C);

  \node  (1)  [left=1cm of A&BvC]  {1.};
  \node  (2)  at (1 |- A)          {2.};
  \node  (3)  at (1 |- BvC)        {3.};
  \node  (4)  at (1 |- B)          {4.};

  \node  (r1) [right=1cm of A&BvC, label=right:{P}]          {};
  \node  (r2) at (r1 |- A)   [label=right:{($\wedge$), 1}]   {};
  \node  (r3) at (r1 |- BvC) [label=right:{($\wedge$), 1}]   {};
  \node  (r4) at (r1 |- B)   [label=right:{($\vee$), 2}]     {};
\end{tikzpicture}
```

You can do the same with numbers, to ensure that they're aligned correctly. But unless you have a lot of numbers, this usually isn't a problem.

§5.2.6 An Illustrative Example

Example 5.14: *Illustrating* `tikzpicture`




```
\begin{tikzpicture}[node distance=1ex]
  \node    (A->B)                                  {$A \rightarrow B$};
  \node    (A&-B)    [below=of A->B]               {$A \wedge \neg B$};
  \node    (-A)      [below left=7mm of A&-B]      {$\neg A$};
  \node    (A)       [below=of -A]                 {$A$};
  \node    (x-A)     [below=of A]                  {$\times$};
  \node    (B)       [below right=7mm of A&-B]     {$B$};
  \node    (A2)      at (x-A -| B) [yshift=-7mm]   {$A$};
  \node    (-B)      [below=of A2]                 {$\neg B$};
  \node    (x-B)     [below=of -B]                 {$\times$};

  \path    (A&-B)    edge[-]    (-A);
  \path    (A&-B)    edge[-]    (B);
  \path    (B)       edge[-]    (A2);

  \node    (1)    [left=2cm of A->B]    {1.};
  \node    (2)    at (1 |- A&-B)        {2.};
  \node    (3)    at (1 |- -A)          {3.};
  \node    (4)    at (1 |- A)           {4.};
  \node    (5)    at (1 |- A2)          {5.};
  \node    (6)    at (1 |- -B)          {6.};

  \path    (5)    edge[-,dashed]    (A2);
  \path    (6)    edge[-,dotted]    (-B);

  \node    (r1)   [right=2cm of A->B, label=right:{P}]                {};
  \node    (r2)   at (r1 |- A&-B) [label=right:{P}]                   {};
  \node    (r3)   at (r1 |- -A)   [label=right:{($\rightarrow$), 1}]  {};
```

```
  \node    (r4)   at (r1 |- A)    [label=right:{($\wedge$), 2}]       {};
  \node    (r5)   at (r1 |- A2)   [label=right:{($\wedge$), 2}]       {};
  \node    (r6)   at (r1 |- -B)   [label=right:{($\wedge$), 2}]       {};
\end{tikzpicture}
```