

Semantic Tableau for Phil 143

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Before we start, make sure to include the following lines in the preamble of your .tex file (before `\begin{document}`):

```
\usepackage{tikz}
\usetikzlibrary{positioning,arrows}

\tikzset{
modal/.style={>=stealth',shorten >=1pt,shorten <=1pt,auto,node distance=1.5cm,
semithick},
tableau/.style={>=stealth',shorten >=1pt,shorten <=1pt,auto,node distance=7mm,
semithick},
world/.style={circle,draw,minimum size=0.5cm,fill=gray!15},
point/.style={circle,draw,inner sep=0.5mm,fill=black},
reflexive above/.style={->,loop,looseness=7,in=120,out=60},
reflexive below/.style={->,loop,looseness=7,in=240,out=300},
reflexive left/.style={->,loop,looseness=7,in=150,out=210},
reflexive right/.style={->,loop,looseness=7,in=30,out=330}}

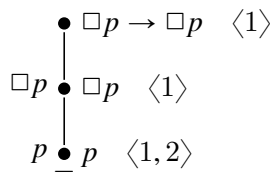
\newcommand{\lab}[1]{\quad \langle #1 \rangle}
```

The command `\lab` will let you add labels in a relatively easy manner.

You can make points in a semantic tableau the same way that you make worlds in modal diagrams. The only difference is that you use point instead of world:

Example (Semantic Tableau)

Showing that $\Box p \rightarrow \Box p$ is valid:



```

\begin{center}
\begin{tikzpicture}[tableau]

\node[point] (1) [label=right:{$\Box p \rightarrow \Box p \ \text{lab}\{1\}$}] {};
\node[point] (2) [label=right:{$\Box p \ \text{lab}\{1\}$},
                 label=left:{$\Box p$},
                 below=of 1] {};
\node[point] (3) [label=right:{$p \ \text{lab}\{1,2\}$},
                 label=left:{$p$},
                 label=below:{$=$},
                 below=of 2] {};

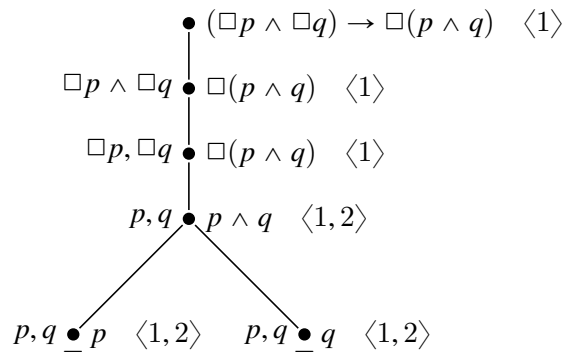
\path[-] (1) edge (2);
\path[-] (2) edge (3);

\end{tikzpicture}
\end{center}

```

Example (Box Validity)

Showing that $(\Box p \wedge \Box q) \rightarrow \Box(p \wedge q)$ is valid:



```

\begin{center}
\begin{tikzpicture}[tableau]

\node[point] (1) [label=right:{$(\Box p \wedge \Box q) \rightarrow \Box(p \wedge q) \ \text{lab}\{1\}$}] {};
\node[point] (2) [label=right:{$\Box(p \wedge q) \ \text{lab}\{1\}$},
                 label=left:{$\Box p \wedge \Box q$},
                 below=of 1] {};

\end{tikzpicture}

```

```

\node[point] (3) [label=right:{$\Box (p \wedge q) \lab{1}$},
                 label=left:{$\Box p, \Box q$},
                 below=of 2] {};
\node[point] (4) [label=right:{$p \wedge q \lab{1,2}$},
                 label=left:{$p, q$},
                 below=of 3] {};
\node[point] (5a) [label=right:{$p \lab{1,2}$},
                  label=left:{$p, q$},
                  label=below:{$=$},
                  below left=2cm of 4] {};
\node[point] (5b) [label=right:{$q \lab{1,2}$},
                  label=left:{$p, q$},
                  label=below:{$=$},
                  below right=2cm of 4] {};

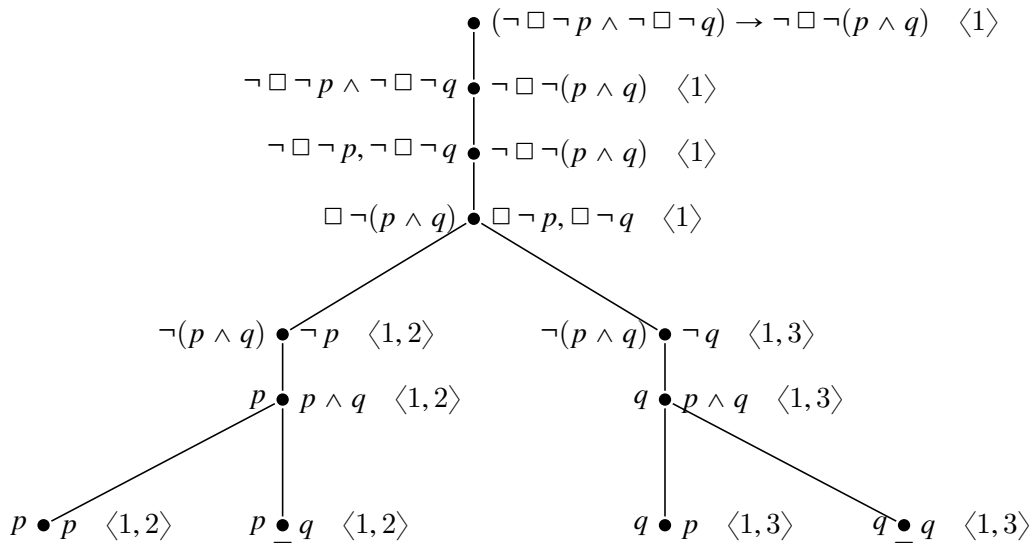
\path[-] (1) edge (2);
\path[-] (2) edge (3);
\path[-] (3) edge (4);
\path[-] (4) edge (5a);
\path[-] (4) edge (5b);

\end{tikzpicture}
\end{center}

```

Example (Diamond Invalidity)

Showing that $(\Diamond p \wedge \Diamond q) \rightarrow \Diamond(p \wedge q)$ is invalid:



```

\begin{center}
\begin{tikzpicture}[tableau]

\node[point] (1) [label=right:{$(\neg\Box\neg p \wedge \neg\Box\neg q) \rightarrow \neg\Box\neg(p \wedge q) \lab{1}$}] {};
\node[point] (2) [label=right:{$\neg\Box\neg(p \wedge q) \lab{1}$},
label=left:{$\neg\Box\neg p \wedge \neg\Box\neg q$},
below=of 1] {};
\node[point] (3) [label=right:{$\neg\Box\neg(p \wedge q) \lab{1}$},
label=left:{$\neg\Box\neg p, \neg\Box\neg q$},
below=of 2] {};
\node[point] (4) [label=right:{$\Box\neg p, \Box\neg q \lab{1}$},
label=left:{$\Box\neg(p \wedge q)$},
below=of 3] {};

\node[point] (5L) [label=right:{$\neg p \lab{1,2}$},
label=left:{$\neg(p \wedge q)$},
below left=2cm of 4,
xshift=-1cm] {};
\node[point] (6L) [label=right:{$p \wedge q \lab{1,2}$},
label=left:{$p$},below=of 5L] {};
\node[point] (7L) [label=right:{$q \lab{1,2}$},
label=left:{$p$},

```

```

        label=below:{$=$},
        below=1.5cm of 6L] {};
\node[point] (8L) [label=right:{$p \lab{1,2}$},
        label=left:{$p$},
        left=3cm of 7L] {};

\node[point] (5R) [label=right:{$\neg q \lab{1,3}$},
        label=left:{$\neg (p \wedge q)$},
        below right=2cm of 4,
        xshift=1cm] {};
\node[point] (6R) [label=right:{$p \wedge q \lab{1,3}$},
        label=left:{$q$},below=of 5R] {};
\node[point] (7R) [label=right:{$p \lab{1,3}$},
        label=left:{$q$},
        below=1.5cm of 6R] {};
\node[point] (8R) [label=right:{$q \lab{1,3}$},
        label=left:{$q$},
        label=below:{$=$},
        right=3cm of 7R] {};

\path[-] (1) edge (2);
\path[-] (2) edge (3);
\path[-] (3) edge (4);
\path[-] (4) edge (5L);
\path[-] (4) edge (5R);
\path[-] (5L) edge (6L);
\path[-] (5R) edge (6R);
\path[-] (6L) edge (7L);
\path[-] (6R) edge (7R);
\path[-] (6L) edge (8L);
\path[-] (6R) edge (8R);

\end{tikzpicture}
\end{center}

```