Equivalences in PPL

October 15, 2013

Throughout, assume α and β are (possibly complex) predicates. We'll use '$\dashv\vDash$' to denote logical equivalence.

Quantifier Exchange:
$\sim\exists\alpha \dashv\vDash \forall\sim\alpha$
$\sim\forall\alpha \dashv\vDash \exists\sim\alpha$

Important Examples:
$\sim\forall(\alpha \supset \beta) \dashv\vDash \exists(\alpha \;\&\; \sim\beta)$
$\sim\exists(\alpha \;\&\; \beta) \dashv\vDash \forall(\alpha \supset \sim\beta)$

Quantifier Replacement:
$\sim\exists\sim\alpha \dashv\vDash \forall\alpha$
$\sim\forall\sim\alpha \dashv\vDash \exists\alpha$

Existence:
$\forall\alpha \vDash \exists\alpha$, **but**:
$\forall(\alpha \supset \beta) \nvDash \exists\alpha$

Conjunction:
$\exists(\alpha \;\&\; \beta) \not\dashv\vDash \exists\alpha \;\&\; \exists\beta$
$\forall(\alpha \;\&\; \beta) \dashv\vDash \forall\alpha \;\&\; \forall\beta$

Disjunction:
$\exists(\alpha \vee \beta) \dashv\vDash \exists\alpha \vee \exists\beta$
$\forall(\alpha \vee \beta) \dashv\nvDash \forall\alpha \vee \forall\beta$

Conditional:
$\forall(\alpha \supset \beta) \not\dashv\vDash \forall\alpha \supset \forall\beta$
$\exists(\alpha \supset \beta) \dashv\vDash \forall\alpha \supset \exists\beta$

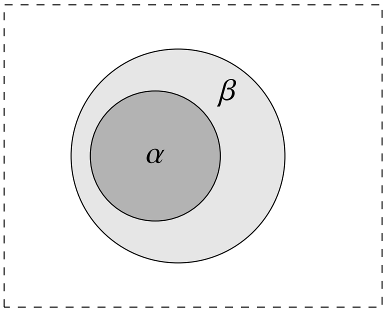
(You can work this out if you remember that $\ulcorner\alpha \supset \beta\urcorner$ is equivalent to $\ulcorner\sim\alpha \vee \beta\urcorner$, and use the rules given.)

Common Translation Schemes from English into PPL:

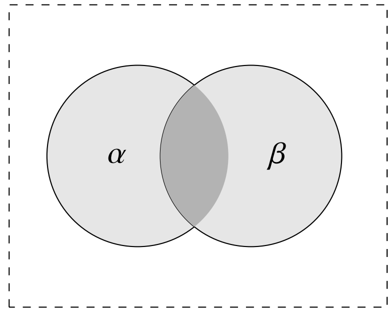
Some αs are βs	$\Rightarrow$	$\exists(\alpha \;\&\; \beta)$
There are some αs that are β	$\Rightarrow$	$\exists(\alpha \;\&\; \beta)$
All αs are βs	$\Rightarrow$	$\forall(\alpha \supset \beta)$
Every α is a β	$\Rightarrow$	$\forall(\alpha \supset \beta)$
No αs are βs	$\Rightarrow$	$\sim\exists(\alpha \;\&\; \beta)$
No αs are βs	$\Rightarrow$	$\forall(\alpha \supset \sim\beta)$
The αs are exactly the βs	$\Rightarrow$	$\forall(\alpha \leftrightarrow \beta)$

Venn Diagram Examples:

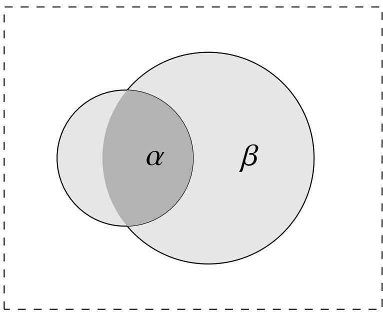
$\forall(\alpha \supset \beta)$:



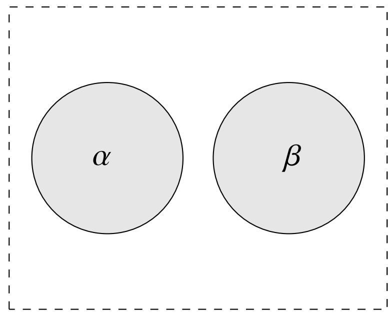
$\exists(\alpha \mathbin{\&} \beta)$:



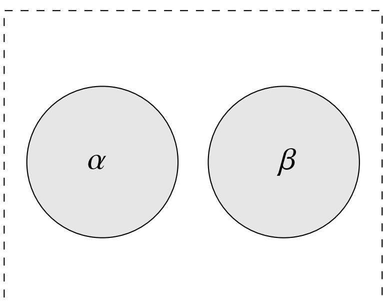
$\sim\forall(\alpha \supset \beta)$:



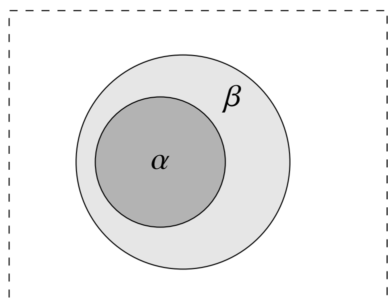
$\sim\exists(\alpha \mathbin{\&} \beta)$:



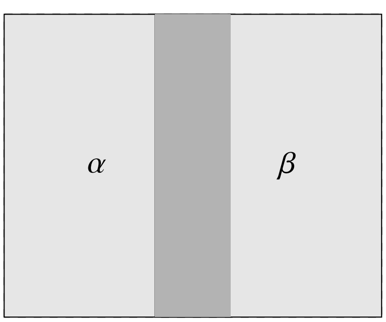
$\forall(\alpha \supset \sim\beta)$:



$\sim\exists(\alpha \mathbin{\&} \sim\beta)$:



$\forall(\alpha \vee \beta)$:



$\exists(\alpha \vee \beta)$: