Worksheet 3 — Feburary 19

1. Fill out the following substitutions, where $S = $ "$p \supset (q \,.\, r)$":

   (a) $S\,[p\,/\,-q] =$

   (b) $S\,[p\,/\,p \supset q] =$

   (c) $S\,[q\,/\,p \vee r] =$

   (d) $S\,[r\,/\,p\,.\,-s] =$

   (e) $S\,[q\,.\,r\,/\,q] =$

   (f) $S\,[p \supset (q\,.\,r)\,/\,s \vee -t] =$

   (g) $S\,[q \vee p\,/\,-s] =$

   (h) $S\,[r\,.\,q\,/\,s \supset p] =$

2. True/False. Don't write out truth tables. (Hint: Use the properties on your handout)

   (a) $\vDash (q\,.\,-r) \vee -q \vee r$

   (b) $(p\,.\,((r \vee -r) \supset q)) \Leftrightarrow p\,.\,q$

   (c) $\vDash\, -p \vee -----p$

   (d) $-p \vDash (p \vee (q \vee -q))$

   (e) If $A \nvDash -B$, then $B \nvDash -A$

   (f) If $A_1, A_2 \vDash B$, then $-B \vDash -A_1 \vee -A_2$

   (g) If $\vDash A \vee B \vee C$, then $-A, -B \vDash C$

   (h) $-A \equiv A \nvDash A$

3. Punctuate the following sentences that need punctuating.

   (a) Underwood is Frank's last name.

   (b) Yoda is a jedi master.

   (c) If Socrates and Plato were philosophers is true, then so are Socrates was a philosopher and Plate was a philosopher.

   (d) If Whitehead and Russell wrote *Principia*, then Whitehead and Russell wrote *Principia* is true is true.

   (e) (b) is false.

   (f) $p\,.\,q \vee r$ is ambiguous.

   (g) If A doesn't imply B, then B implies A.

   (h) A, B, and C are names for syntactic variables.

   (i) $p \vee q \nvDash p\,.\,q$

   (j) $p \supset (q \vee r)$ consists of seven symbols.

   (k) $A\,.\,B \vee C$ is ambiguous.

   (l) A and B imply the disjunction of C and D.

4. For each of the following "general laws," (i) formalize the law, and (ii) determine whether it's true or false. If it's true, prove it. If it's false, find a counter-example. For instance, if I was given:

   If a schema does not imply another schema, then the second schema implies the first.

   I would say:

   (i) Formalization: If $A \nvDash B$, then $B \vDash A$.
   (ii) This law is false. Counter-example: $p \nvDash q$, but $q \nvDash p$.

   (a) If the biconditional of two schemata is valid, then the two schema are equivalent.
   (b) If two schemata imply a third, then the negation of either of the first two schema implies the negation of the third.
   (c) If two schemata imply a third schema, then the negation of the third schema implies the disjunction of the negations of the first two scehmata.
   (d) If a schema implies a disjunction of two other schemata, then the first schema implies one of the other two schemata.
   (e) If a conditional is unsatisfiable, then the antecedent is valid.
   (f) If a disjunction of three schemata is valid, then the negation of the first and the negation of the third together imply the second schema.
   (g) If a disjunction is satisfiable, then at least one of the two disjuncts is satisfiable.
   (h) If a disjunction is valid, the at least one of the two disjuncts is valid.
   (i) If a conditional is satisfiable, then the consequent is satisfiable.
   (j) If one schema implies the conditional of two schemata, then the second schema implies the conditional of the first and third schemata.
   (k) If a valid conditional has as its consequent the antecedent of another valid conditional, then the conditional with the antecedent of the first and the consequent of the second is valid.