Worksheet 4 — April 2

1. Schematize the following (define your own predicates explicitly):

   (i) Every jedi has been trained by someone.

   (ii) Someone has trained all of the jedi.

   (iii) Some secret agent has everyone's contact information.

   (iv) No student is smarter than every student.

2. Schematize the following sentences. Use the following predicate letters with their intended interpretations:

$$\begin{array}{lcl} D & = & \text{“ ① defeated ② ”} \\ E & = & \text{“ ① is the enemy of ② ”} \\ F & = & \text{“ ② decides the fate of ① ”} \\ G & = & \text{“ ① is a gladiator”} \\ W & = & \text{“ ③ watched ① fight ② ”} \end{array}$$

   (i) Every gladiator has defeated another gladiator.

   (ii) No gladiator has defeated him/herself.[1]

   (iii) Someone decides the fate of every gladiator. (Two readings; schematize both)

   (iv) Everyone has an enemy, but no one is his/her own enemy.

   (v) If someone's enemies has enemies, then they are that person's enemies as well.

   (vi) Every gladiator has defeated everyone that was defeated by someone he has defeated.

   (vii) x was defeated by one of his/her enemies.

   (viii) If x is the enemy of y, then x decides the fate of y provided that x defeated y.

   (ix) x decides the fate of y iff x watched y fight someone that defeated y.

   (x) Someone decides the fate of all and only those who doesn't decide their own fate.[2]

   (xi) Every gladiator who was watched fighting their enemy decided the fate of another gladiator they watched their enemy fight.

   (xii) No gladiator watched someone who decided their fate fight someone else.

[1] Fun fact: did you know there were female gladiators? Technically, the term is "gladiatrix".
[2] Is there something weird about this sentence?

3. Translate the following schemata into clear, idiomatic English.

   Example: Translate: $\forall x\,(Fx \supset \exists y\, Rxy)$

   "Every object is such that if it is F, there is something that it R's"

   i.e. "Everything that's F also R's something."

   (i) $\forall x\,(Fx \supset \forall y\, Rxy)$

   (ii) $\exists x\,(Fx\,.\,\forall y\, Rxy)$

   (iii) $\forall x \exists y \forall z\,(Ryz \supset Sxz)$

   (iv) $\exists x \forall y (Rxy) \supset \forall x \exists y (Ryx)$

   (v) $\forall x \forall y (Rxy\,.\,Fy \supset Gx) \vee \exists x Hxx$

   (vi) $\exists x \forall y\,(Rxy \equiv -Rxx)$

4. (For the math aficionados): Given the standard interpretation of the symbols below, what do the following sentences say, and what mathematical concept(s) are they describing? (Note: for some relation symbols, we use *infix* notation, i.e. we write "xRy" instead of "Rxy")

   (a) $\forall x \forall y \exists z\,(x < z\,.\,y < z)$, $\forall x \forall y \exists z\,(z < x\,.\,z < y)$

   (b) $\forall x \forall y \exists z\,(z|x\,.\,z|y\,.\,\forall u\,(u|x\,.\,u|y \supset u|z))$

   (c) $\forall x(x \sim x)\,.\,\forall x \forall y(x \sim y \supset y \sim x)\,.\,\forall x \forall y \forall z(x \sim y\,.\,y \sim z \supset x \sim z)$

   (d) $\forall x(-(x < x))\,.\,\forall x \forall y\,((x < y) \vee (y < x) \vee (x = y))\,.\,\forall x \forall y \forall z(x < y\,.\,y < z \supset x < z)$

   (e) $\forall x \forall y \forall z\,((x \cdot y) \cdot z = x \cdot (y \cdot z))\,.\,\forall x(x \cdot 1 = x)\,.\,\forall x(x \cdot x^{-1} = 1)$ [3]

   Can you think of a mathematical concept that cannot be completely described by a first-order sentence? (I'm thinking of a rather basic concept)

[3] Note: we haven't technically introduced *function symbols* in our language, like "$\cdot$" or "$^{-1}$"; nor have we introduced *constants* like "1." But we can treat all such instances of functions and constants as special kinds of *relations*, so no harm is done importing them into our language here.