Worksheet 5 — April 11

1. For each schema below, use the set $\{1, 2, 3\}$ as your UD. In each case: (a) find an interpretation that makes the schema true, (b) find an interpretation that makes the schema false, and (c) draw an arrow diagram of each interpretation. You *cannot* assign R to $\varnothing$.

   (i) $\exists x \forall y (Ryx \supset Ryy)$

   (ii) $\forall x \forall y \, (\exists z (Rxz \,.\, Ryz) \supset Rxy)$

   (iii) $\forall x \forall y \, (Rxy \supset \exists z (Rxz \,.\, Rzy))$

   (iv) $\exists x \exists y \, (Rxy \,.\, Ryx) \;.\; \forall x \forall y \, (\exists z \, (Rxz \,.\, Rzy) \supset Rxy)$

   (v) $\exists x \forall y Rxy \,.\, \forall x \, (\forall y Rxy \supset \forall y Ryx)$

   (vi) $\forall x \forall y Rxy$

2. Let v be an interpretation of a binary predicate R. We say that R on v is:

   - **reflexive** if "$\forall x Rxx$" is true
   - **irreflexive** if "$\forall x - Rxx$" is true
   - **symmetric** if "$\forall x \forall y (Rxy \supset Ryx)$" is true
   - **asymmetric** if "$\forall x \forall y (Rxy \supset -Ryx)$" is true
   - **antisymmetric** if "$\forall x \forall y (Rxy \,.\, Ryx \supset x = y)$" is true[1]
   - **transitive** if "$\forall x \forall y \forall z \, (Rxy \,.\, Ryz \supset Rxz)$" is true
   - **euclidean** if "$\forall x \forall y \, (\exists z \, (Rzx \,.\, Rzy) \supset Rxy)$" is true

   Find a counterexample to each implication below. You may either give an interpretation or draw an arrow diagram. (These can all be done with UD = $\{1, 2, 3\}$)

   (i) symmetric $\Rightarrow$ reflexive

   (ii) antisymmetric $\Rightarrow$ asymmetric

   (iii) euclidean $\Rightarrow$ symmetric

   (iv) symmetric + transitive $\Rightarrow$ reflexive

3. Argue for the following.

   (i) antisymmetric + irreflexive $\Rightarrow$ asymmetric

   (ii) asymmetric $\Rightarrow$ antisymmetric

   (iii) reflexive + symmetric + transitive $\Leftrightarrow$ reflexive + euclidean

   (iv) transitive + irreflexive $\Rightarrow$ asymmetric

[1] The interpretation of "=" is fixed: it is always the identity relation on any interpretation.