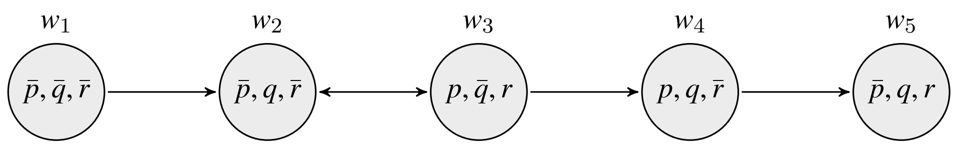
Counterfactuals

Phil 143 Worksheet

1. Using the truth definition for counterfactuals on slide 13, determine which of the following formulas below is true at $\mathcal{M}, w_1$.



The relevant ordering is given as follows:

w_1: $w_1 < w_2 \approx w_3 < w_4 < w_5$
w_3: $w_3 < w_2 < w_1 < w_4 < w_5$ (not drawn, only needed for (d))

(a) $p \mathrel{\Box\!\!\rightarrow} r$

(b) $(p \wedge q) \mathrel{\Box\!\!\rightarrow} r$

(c) $\neg p \mathrel{\Box\!\!\rightarrow} (p \mathrel{\Box\!\!\rightarrow} q)$

(d) $p \mathrel{\Box\!\!\rightarrow} (\neg p \mathrel{\Box\!\!\rightarrow} r)$

2. Show that $(p \mathrel{\Box\!\!\rightarrow} q) \vee (p \mathrel{\Box\!\!\rightarrow} \neg q)$ is valid on a world-ordering frame $\mathcal{F} = \langle W \{\leqslant_w\}_{w \in W} \rangle$ iff Stalnaker's assumption holds on $\mathcal{F}$: for all nonempty $S \subseteq W$ and all $w \in W$, $|\mathbf{Min}_{\leqslant_w}(S)| = 1$.

3. For each formula below, determine whether or not that formula is valid on Lewis's semantics. If the formula is valid, prove it by showing that every pointed model which makes the antecedent true makes the consequent true. If the formula is not valid, construct a pointed model that falsifies it. (Assume the ordering relations are well-founded and total, so that you can use the truth definition on slide 13. Also assume every $\leqslant_w$ is weakly centered.)

(a) $\alpha \mathrel{\Box\!\!\rightarrow} (\beta \mathrel{\Box\!\!\rightarrow} \alpha)$

(b) $((\alpha \wedge \beta) \mathrel{\Box\!\!\rightarrow} \gamma) \rightarrow (\alpha \mathrel{\Box\!\!\rightarrow} (\beta \mathrel{\Box\!\!\rightarrow} \gamma))$

(c) $(((\alpha \wedge \beta) \mathrel{\Box\!\!\rightarrow} \gamma) \wedge (\alpha \mathrel{\Box\!\!\rightarrow} \beta)) \rightarrow (\alpha \mathrel{\Box\!\!\rightarrow} \gamma)$

(d) $(\alpha \mathrel{\Box\!\!\rightarrow} (\beta \mathrel{\Box\!\!\rightarrow} \gamma)) \rightarrow (\beta \mathrel{\Box\!\!\rightarrow} (\alpha \mathrel{\Box\!\!\rightarrow} \gamma))$

(An aside: notice that replacing $\Box\!\!\rightarrow$ in the formulas throughout results in propositional tautologies. Similarly for the problems and model answer on the problem set, and for 1(c)–(d) and 2 above. This hopefully illustrates that very few $\rightarrow$-validities carry over to $\Box\!\!\rightarrow$.)