

# Extensions of K

## Phil 143 Worksheet

1. Produce axiomatic proofs that show the following:
  - (a)  $\vdash_{\mathbf{KT5}} \varphi \rightarrow \Box \Diamond \varphi$
  - (b)  $\vdash_{\mathbf{KB4}} \Box(\Diamond \varphi \rightarrow \psi) \rightarrow \Box(\varphi \rightarrow \Box \psi)$
  - (c)  $\vdash_{\mathbf{KD4}} \Box \Diamond \varphi \rightarrow \Diamond \varphi$
  - (d)  $\vdash_{\mathbf{K5}} \Diamond \Diamond \varphi \rightarrow \Box \Diamond \varphi$
2. Let  $\mathcal{F} = \langle W, R \rangle$  be a frame. Prove that  $\Diamond p \rightarrow \Diamond \Diamond p$  is valid on  $\mathcal{F}$  iff  $R$  is *dense*:  
 $\forall w, v \in W$  if  $wRv$ , then  $\exists u \in W$  such that  $wRu$  and  $uRv$ .
3. Find the frame correspondent for  $\Diamond(p \rightarrow \Diamond p)$ .
4. Show that  $R^{\mathbf{K4}}$  is transitive:  $\forall \Gamma_1, \Gamma_2, \Gamma_3$  if  $\Gamma_1 R^{\mathbf{K4}} \Gamma_2$  and  $\Gamma_2 R^{\mathbf{K4}} \Gamma_3$ , then  $\Gamma_1 R^{\mathbf{K4}} \Gamma_3$ .