Modal Logic Symbols

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This is a brief guide to typing symbols from modal logic. If you've written a paper on modal logic, you've probably used `\Box` and `\Diamond` as your go-to modal operators. But stacked next to each other, they don't look that great:

$$\Box\Diamond\Box\phi$$

The issue (in my opinion) is that $\Box$ and $\Diamond$ are very different in size and so look squished together. To solve this, I've created my own versions where the box and diamond:

$$\boxempty\, \diamondsuit\, \phi$$

This guide also presents a greater variety of modal operators:

$$\boxempty, \diamondsuit, \bigcirc, \triangle, \triangledown, \triangleleft, \triangleright, \bigstar$$

As well as additional fillings for $\boxempty$, $\diamondsuit$, and $\bigcirc$. For example, here are some options for $\boxempty$:

$$\blacksquare, \boxdot, \boxminus, \boxplus, \boxtimes$$

I've also included some commonly used operators, like the counterfactual arrow $\boxrightarrow$. Furthermore, I've made several counterfactual "biconditional" arrows, which I couldn't find defined in any other package:

$$\boxleftarrow, \boxLeftarrow, \boxdotleftarrow, \boxdotLeftarrow$$
$$\diamondleftarrow, \diamondLeftarrow, \circdiamondleftarrow, \circdiamondLeftarrow$$

I've collected the commands for all of these symbols into a package file called `modalops.sty`. You can download it here.

1 Modal Operator Commands

Below is a list of modal operator commands. The commands for binary operators are all defined to have the form `\mathbin{...}`. This ensures that spacing around the symbol works as expected. Similarly, the commands for unary operators are all defined to have the form:

```
\mathop{...}\nolimits
```

The `\nolimits` ensures that subscripts and superscripts are placed next to the operator, rather than above or below it (as illustrated below). If you want this effect, you can simply delete the `\nolimits` from the relevant commands in the provided .sty file.

`\nec_1\phi` defined with `\nolimits` $\Rightarrow$ $\boxempty_1 \phi$

`\nec_1\phi` defined without `\nolimits` $\Rightarrow$ $\underset{1}{\boxempty}\, \phi$

Standard Modal Operators

□	\nec	◇	\pos	○	\deo	△	\tri	☆	\star
■	\necf	◆	\posf	●	\deof	▲	\trif	★	\starf
⊡	\necd	⟐	\posd	⊙	\deod	▽	\trid	✶	\starv
⊟	\necm	⟑	\posm	⊖	\deom	▼	\tridf	✡	\starp
◫	\necv	⟡	\posv	⦶	\deov	◁	\tril	✡	\stard
⊞	\necp	⟠	\posp	⊕	\deop	◀	\trilf		
⊠	\necx	⟢	\posx	⊗	\deox	▷	\trif		
						▶	\trirf		

Counterfactuals and Strict Conditionals

□→	\necif	←□	\necfi	←□→	\neciff
⊡→	\necdif	←⊡	\necdfi	←⊡→	\necdiff
□⇒	\necIf	⇐□	\necFi	⇐□⇒	\necIff
⊡⇒	\necdIf	⇐⊡	\necdFi	⇐⊡⇒	\necdIff
◇→	\posif	←◇	\posfi	←◇→	\posiff
⟐→	\posdif	←⟐	\posdfi	←⟐→	\posdiff
◇⇒	\posIf	⇐◇	\posFi	⇐◇⇒	\posIff
⟐⇒	\posdIf	⇐⟐	\posdFi	⇐⟐⇒	\posdIff
○→	\deoif	←○	\deofi	←○→	\deoiff
⊙→	\deodif	←⊙	\deodfi	←⊙→	\deodiff
⇾	\strictif	⇽	\strictfi	⇽⇾	\strictiff

2 List of Primitive Symbols

If you just want the command for a symbol, without being enclosed in a modal operator environment (e.g., \modalop), here is a complete table of all the primitive symbols and their commands.

□	\medsquare	◇	\meddiamond
■	\medsquarefilled	◆	\meddiamondfilled
⊡	\medsquaredot	⟐	\meddiamonddot
⊟	\medsquareminus	⟑	\meddiamondminus
◫	\medsquarevert	⟡	\meddiamondvert
⊞	\medsquareplus	⟠	\meddiamondplus
⊠	\medsquaretimes	⟢	\meddiamondtimes
○	\medcircle	▷	\medtriangleright
●	\medcirclefilled	▶	\medtrianglerightfilled
⊙	\medcircledot	◁	\medtriangleleft
⊖	\medcircleminus	◀	\medtriangleleftfilled
⦶	\medcirclevert	△	\medtriangleup
⊕	\medcircleplus	▲	\medtriangleupfilled
⊗	\medcircletimes	▽	\medtriangledown
		▼	\medtriangledownfilled

☆ \medstar
★ \medstarfilled
✶ \medstarvar

✡ \medpentagram
✡ \medstarofdavid

□→ \boxrightarrow
⊡→ \boxdotrightarrow
□⇒ \boxRightarrow
⊡⇒ \boxdotRightarrow
←□ \boxleftarrow
←⊡ \boxdotleftarrow
⇐□ \boxLeftarrow
⇐⊡ \boxdotLeftarrow
←□→ \boxleftrightarrow
⇐□⇒ \boxLeftrightarrow
←⊡→ \boxdotleftrightarrow
⇐⊡⇒ \boxdotLeftrightarrow

◇→ \diamondrightarrow
⟐→ \diamonddotrightarrow
◇⇒ \diamondRightarrow
⟐⇒ \diamonddotRightarrow
←◇ \diamondleftarrow
←⟐ \diamonddotleftarrow
⇐◇ \diamondLeftarrow
⇐⟐ \diamonddotLeftarrow
←◇→ \diamondleftrightarrow
⇐◇⇒ \diamondLeftrightarrow
←⟐→ \diamonddotleftrightarrow
⇐⟐⇒ \diamonddotLeftrightarrow

○→ \circlerightarrow
⊙→ \circledotrightarrow
←○ \circleleftarrow
←⊙ \circledotleftarrow
←○→ \circleleftrightarrow
←⊙→ \circledotleftrightarrow

↣ \rightfishhook
↢ \leftfishhook
↢↣ \leftrightfishhook